Plošné integrály - 2002

1. Plošné integrály funkce.

1. $\iint_S xz\,\mathrm{d}S$; $S = \{(x,y,z);\ x+y+z=1,\ x>0,\ y>0,\ z>0\}$. $[\frac{\sqrt{3}}{24}]$.
2. $\iint_S x\,\mathrm{d}S$; $S = \{(x,y,z);\ x+y+z=1,\ x>0,\ y>0,\ z>0\}$. $[\frac{\sqrt{3}}{6}]$.
3. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ z=xy,\ x^2+y^2 \le 4\}$. $[\frac{2\pi}{3}(5\sqrt{5}-1)]$.
4. $\iint_S |xy|\mathrm{d}S$; $S = \{(x,y,z);\ z=xy,\ x^2+y^2 \le 1\}$. $[\frac{4}{15}(\sqrt{2}+1)]$.
5. $\iint_S |xy|\mathrm{d}S$; $S = \{(x,y,z);\ z=x^2+y^2,\ |x|<1,\ |y|<2\}$. $[\frac{1}{240}(441\sqrt{21}-289\sqrt{17}-25\sqrt{5}+1)]$.
6. $\iint_S (x^2+y^2)\mathrm{d}S$; $S = \{(x,y,z);\ z=x^2+y^2,\ z<1\}$. $[\frac{\pi}{60}(25\sqrt{5}+1)]$.
7. $\iint_S (x^2+z^2)\mathrm{d}S$; $S = \{(x,y,z);\ z^2=x^2+y^2,\ 0<z<1,\ y>0\}$. $[\frac{3\pi\sqrt{2}}{8}]$.
8. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ z=\sqrt{x^2+y^2},\ x^2+y^2<2y\}$. $[\pi\sqrt{2}]$.
9. $\iint_S |xy|\mathrm{d}S$; $S = \{(x,y,z);\ z=\sqrt{x^2+y^2},\ x^2+y^2<2x\}$. $[\frac{4\sqrt{2}}{3}]$.
10. $\iint_S z\,\mathrm{d}S$; $S = \{(x,y,z);\ z=\sqrt{x^2+y^2},\ x^2+y^2<-2y\}$. $[\frac{32\sqrt{2}}{9}]$.
11. $\iint_S xy\,\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2=4,\ x>0,\ y>0,\ 0<z<3\}$. $[12]$.
12. $\iint_S (2x+3y-z+4)\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2=4,\ x>0,\ y>0,\ 0<z<3\}$. $[60+\frac{15\pi}{2}]$.
13. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ z=4-x^2+y^2,\ x^2+y^2<9\}$. $[\frac{\pi}{6}(37\sqrt{37}-1)]$.
14. $\iint_S (x^2+y^2)\mathrm{d}S$; $S = \{(x,y,z);\ z=4-x^2+y^2,\ x^2+y^2<1\}$. $[\frac{\pi}{60}(25\sqrt{5}+1)]$.
15. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2=4,\ z>0\}$. $[8\pi]$.
16. $\iint_S z\,\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2=4,\ z>0\}$. $[8\pi]$.
17. $\iint_S |xy|\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2=4,\ z>0\}$. $[\frac{64}{3}]$.
18. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+6z=6,\ x>0,\ y>0,\ z>0\}$. $[\frac{\cancel{21}}{\cancel{8}}]$
19. $\iint_S (x+y+z)\mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+z=6,\ x>0,\ y>0,\ z>0\}$. $[11\sqrt{14}]$.
20. $\iint_S xyz\,\mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+z=6,\ x^2+y^2<1\}$. $[0]$.
21. $\iint_S (x^2+y^2)\mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+z=6,\ x^2+y^2<1\}$. $[\frac{\pi\sqrt{14}}{2}]$.
22. $\iint_S (x+y)\mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+z=6,\ x^2+y^2<1,\ x>0,\ y>0\}$. $[\frac{2\sqrt{14}}{3}]$.
23. $\iint_S (x^2+y^2+z^2)\mathrm{d}S$; $S = \{(x,y,z);\ 2x+3y+z=6,\ x^2+y^2<1,\ x>0,\ y>0\}$. $[\sqrt{14}(\frac{159\pi}{16}-20)]$.
24. $\iint_S z\,\mathrm{d}S$; $S = \{(x,y,z);\ z=xy,\ x>0,\ y>0,\ x^2+y^2<1\}$. $[\frac{1+\sqrt{2}}{15}]$.
25. $\iint_S (x^2+y^2+z^2)\mathrm{d}S$; $S = \{(x,y,z);\ z=xy,\ x>0,\ y>0,\ x^2+y^2<1\}$. $[\frac{3\pi}{140}(1+\sqrt{2})]$.
26. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ z=\sqrt{x^2+y^2},\ x>0,\ y>0,\ z<1\}$. $[\frac{\pi\sqrt{2}}{4}]$.
27. $\iint_S z\,\mathrm{d}S$; $S = \{(x,y,z);\ z=\sqrt{x^2+y^2},\ x>0,\ y>0,\ z<1\}$. $[\frac{\pi\sqrt{2}}{6}]$.

28. $\iint_S \dfrac{1}{\sqrt{x^2+y^2+z^2}}\mathrm{d}S$; $S = \{(x,y,z);\ z = \sqrt{x^2+y^2},\ x > 0,\ y > 0\}$. $[\frac{\pi}{2}]$.

29. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 2(x^2+y^2) < 1\}$. $[\pi(2-\sqrt{2})]$.

30. $\iint_S z\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 2(x^2+y^2) < 1\}$. $[\frac{\pi}{2}]$.

31. $\iint_S (x^2+y^2)\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 2(x^2+y^2) < 1\}$. $[\frac{\pi}{6}(8-5\sqrt{2})]$.

32. $\iint_S \mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 0 < z < \sqrt{x^2+y^2}\}$. $[\pi\sqrt{2}]$.

33. $\iint_S z\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 0 < z < \sqrt{x^2+y^2}\}$. $[\frac{\pi}{2}]$.

34. $\iint_S (x^2+y^2)\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 0 < z < \sqrt{x^2+y^2}\}$. $[\frac{5\pi\sqrt{2}}{6}]$.

35. $\iint_S \dfrac{1}{x^2+y^2}\mathrm{d}S$; $S = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ 0 < z < \sqrt{x^2+y^2}\}$. $[\pi\ln(3+\sqrt{2})]$.

36. $\iint_B xy\,\mathrm{d}S$; $B = \{(x,y,z);\ z = \sqrt{x^2+y^2},\ x^2+y^2 \le x,\ y > 0\}$ $[\frac{\sqrt{2}}{24}]$

37. $\iint_B (x^2+y^2)\,\mathrm{d}S$; $B = \{(x,y,z);\ z^2 = x^2+y^2,\ -1 \le z \le 2\}$ $[\frac{17\pi}{2}\sqrt{2}]$

38. $\iint_B x\,\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ z > 0,\ x > 0\}$ $[\frac{\pi}{2}]$

39. $\iint_B xz\,\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2 = 1,\ 0 < z < 2,\ x > 0\}$ $[4]$

40. $\iint_B xy\,\mathrm{d}S$; $B = \{(x,y,z);\ z^2 = x^2+y^2,\ 0 < z < 1,\ x > 0,\ y > 0\}$ $[\frac{\sqrt{2}}{8}]$

41. $\iint_B \mathrm{d}S$; $B = \{(x,y,z);\ z = xy,\ x^2+y^2 \le 4\}$ $[\frac{2\pi}{3}(5\sqrt{5}-1)]$

42. $\iint_B \dfrac{1}{(1+x+y)^2}\mathrm{d}S$; $B = \{(x,y,z);\ x+y+z \le 1,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$ $[\sqrt{3}(\ln 2 - \frac{1}{2})]$

43. $\iint_B \mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2+z^2 = 9,\ x^2+y^2 \le 3y,\ z > 0\}$ $[9(\pi-2)]$

44. $\iint_B |xyz|\mathrm{d}S$; $B = \{(x,y,z);\ z = x^2+y^2,\ z < 1\}$ $[\frac{1}{4}(\frac{125\sqrt{5}-1}{105})]$

45. $\iint_B z\mathrm{d}S$; $B = \{(x,y,z);\ x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = \varphi,\ 0 < \rho \le 1,\ 0 \le \varphi \le 2\pi\}$ $[\pi^2(\sqrt{2}+\ln(1+\sqrt{2}))]$

46. $\iint_B (xy+yz+xz)\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2 = z^2,\ x^2+y^2 \le 2x,\ z \ge 0\}$ $[\frac{64\sqrt{2}}{15}]$

47. $\iint_B (x+y+z)\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2+z^2 = 4,\ z \ge 0\}$ $[8\pi]$

48. $\iint_B \sqrt{1-x^2-y^2}\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2+z^2 = 1,\ x \ge 0,\ y \ge 0,\ z \ge 0,\ x+y \le 1\}$ $[\frac{1}{2}]$

49. $\iint_B \dfrac{1}{x^2+y^2+z^2}\mathrm{d}S$; $B = \{(x,y,z);\ x^2+y^2 = 1,\ 0 \le z \le 1\}$ $[\frac{\pi^2}{2}]$

50. $\iint_S (x+y+z)\mathrm{d}S$; S je hranice $\langle 0,1\rangle \times \langle 0,1\rangle \times \langle 0,1\rangle$. $[9]$

51. $\iint_B \mathrm{d}S$; $B = \{(x,y,z);\ 2x+y-z = 0,\ \dfrac{x^2}{9}+\dfrac{y^2}{16} \le 1\}$ $[12\sqrt{6}\pi]$

2. Plošné integrály vektorových polí.

52. $\iint_{(S)} xz\,\mathrm{d}x\mathrm{d}y$; $S = \{(x,y,z);\ x+y+z = 1,\ x > 0,\ y > 0,\ z > 0\}$ $[\frac{1}{24}]$

53. $\iint_{(S)} y\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z$; $S = \{(x,y,z);\ x+z = 1,\ x^2+y^2 \le 1\}$ $[0]$

54. $\iint_{(S)} x\,\mathrm{d}y\mathrm{d}z + y\,\mathrm{d}z\mathrm{d}x + z\,\mathrm{d}x\mathrm{d}y$; $S = \{(x,y,z);\ \frac{x^2}{4}+y^2+\frac{z^2}{9} = 1,\ x \ge 0,\ y \ge 0,\ z \ge 0\}$ $[3\pi]$

55. $\iint_{(B)} x\,\mathrm{d}y\mathrm{d}z + y\,\mathrm{d}z\mathrm{d}x + z\,\mathrm{d}x\mathrm{d}y$; $B = \{(x,y,z);\ x^2+y^2 = 1,\ 0 \le z \le 1\}$ $[3\pi]$

56. $\iint_{(B)} xy\,\mathrm{d}x\mathrm{d}z;$ $\quad B = \{(x,y,z);\ z = x^2+y^2,\ x>0,\ y>0,\ z<1\}$ $\quad [\frac{-2}{15}]$

57. $\iint_{(B)} xy^2\,\mathrm{d}y\mathrm{d}z;$ $\quad B = \{(x,y,z);\ x+y+z=1,\ x>0,\ y>0,\ z>0\}$ $\quad [\frac{1}{60}]$

58. $\iint_{(B)} x\mathrm{d}y\mathrm{d}z + y\mathrm{d}x\mathrm{d}z + (z^2-1)\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ x^2+y^2=1,\ 0\le z\le 1\}$ $\quad [2\pi]$

59. $\iint_{(B)} x\mathrm{d}y\mathrm{d}z + y\mathrm{d}x\mathrm{d}z;$ $\quad B = \{(x,y,z);\ x^2+y^2+z^2=4,\ z\ge 0\}$ $\quad [\frac{32\pi}{3}]$

60. $\iint_{(B)} y\mathrm{d}y\mathrm{d}z + z\mathrm{d}x\mathrm{d}z + x^2\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ x^2+y^2=z^2,\ 0\le z\le 2\}$ $\quad [-4\pi]$

61. $\iint_{(B)} \mathrm{d}y\mathrm{d}z + z^2\mathrm{d}x\mathrm{d}z;$ $\quad B = \{(x,y,z);\ z^2 = x^2-y^2,\ x^2+y^2\le 1\}$ $\quad [0]$

62. $\iint_{(B)} x^2y^2z\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ 4x^2+y^2+z^2=1,\ z>0\}$ $\quad [\frac{\pi}{420}]$

63. $\iint_{(B)} z\mathrm{d}x\mathrm{d}y - (x+y)\mathrm{d}z\mathrm{d}x;$ $\quad B = \{(x,y,z);\ z = x^2+y^2,\ 0\le z\le 1\}$ $\quad [\pi]$

64. $\iint_{(B)} xz\mathrm{d}y\mathrm{d}z + x^2y\mathrm{d}z\mathrm{d}x + y^2z\mathrm{d}x\mathrm{d}y;$ $B = \{(x,y,z);\ x^2+y^2=1,\ x\ge 0,\ y\ge 0,\ 0\le z\le 1\}$ $\quad [\frac{3\pi}{16}]$

65. $\iint_{(S)} x^2\mathrm{d}y\mathrm{d}z + y^2\mathrm{d}z\mathrm{d}x + z^2\mathrm{d}x\mathrm{d}y;$ $\quad S$ je hranice $\langle 0,1\rangle \times \langle 0,1\rangle \times \langle 0,1\rangle$ $\quad [3]$

66. $\iint_{(B)} xz\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ x+y+z=1,\ x\ge 0,\ y\ge 0,\ z\ge 0\}$ $\quad [\frac{1}{24}]$

67. $\iint_{(B)} y\mathrm{d}y\mathrm{d}z + z\mathrm{d}z\mathrm{d}x;$ $\quad B = \{(x,y,z);\ x+z=1,\ x^2+y^2\le 1\}$ $\quad [0]$

68. $\iint_{(B)} x\mathrm{d}y\mathrm{d}z + y^2\mathrm{d}z\mathrm{d}x + yz\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ x^2+y^2=(z-1)^2,\ 0\le z\le 1\}$ $\quad [\frac{\pi}{3}]$

69. $\iint_{(B)} y^2\mathrm{d}z\mathrm{d}x + z\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ z = xy,\ x^2+y^2\le 1,\ x\ge 0,\ y\ge 0\}$ $\quad [\frac{7}{120}]$

70. $\iint_{(B)} y^2\mathrm{d}z\mathrm{d}x + z\mathrm{d}x\mathrm{d}y;$ $\quad B = \{(x,y,z);\ z = x^2-y^2,\ 0\le x\le 1,\ |y|\le 1\}$ $\quad [0]$