

Základní úlohy Mongeovho promítání

Marek Pivovarník

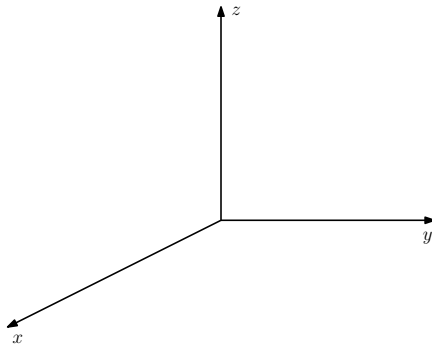
ÚM FSI VUT v Brně

12. 11. 2013

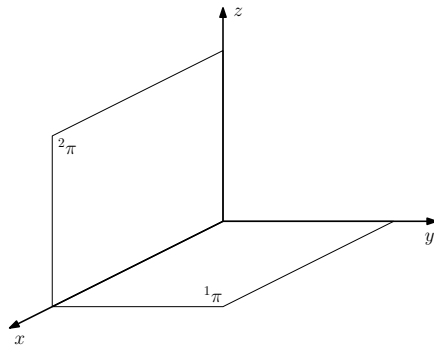
1.

BOD v MP

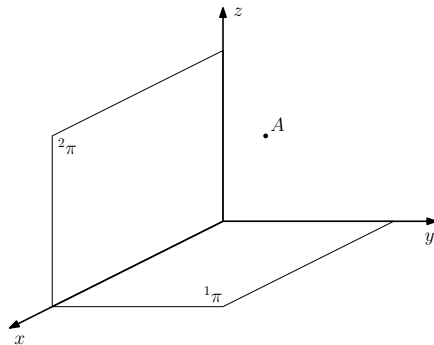
Bod



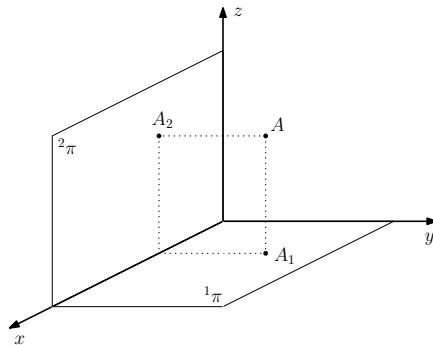
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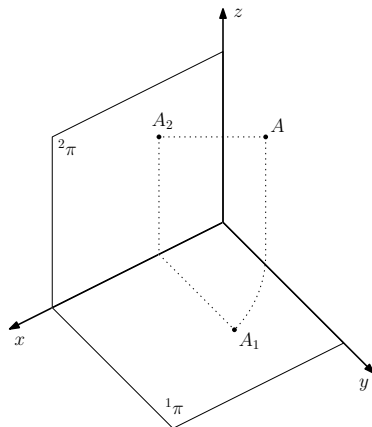
Bod



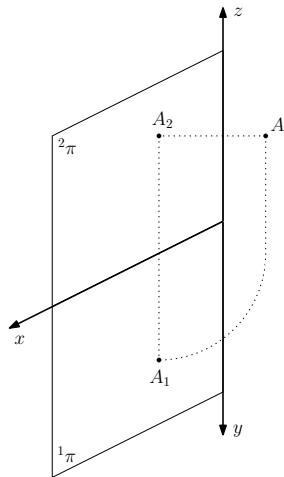
Bod



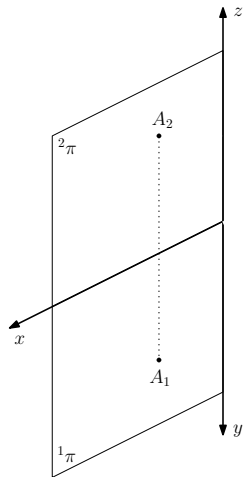
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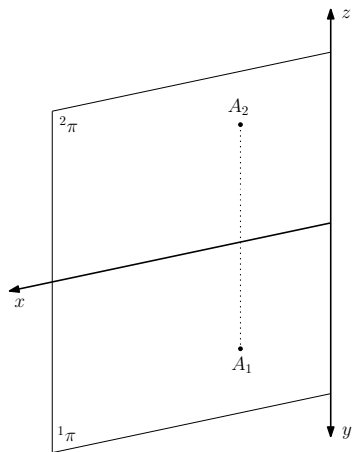
Bod



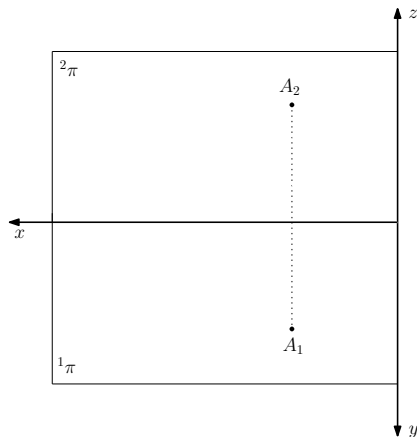
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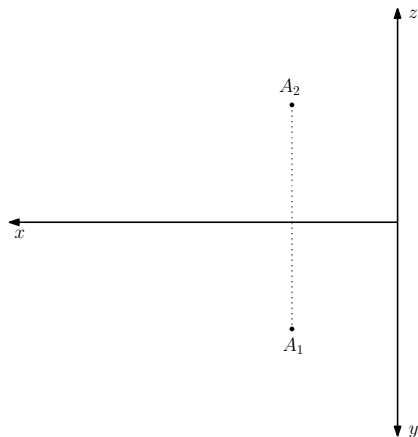
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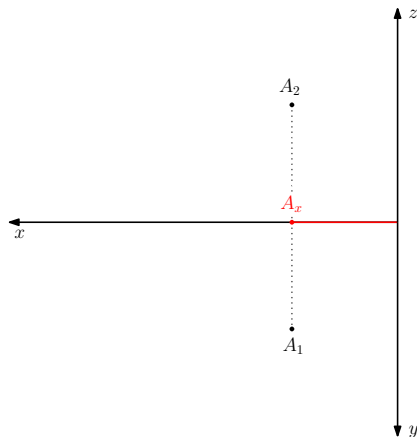
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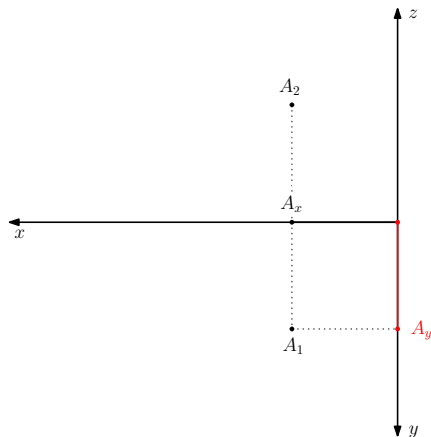
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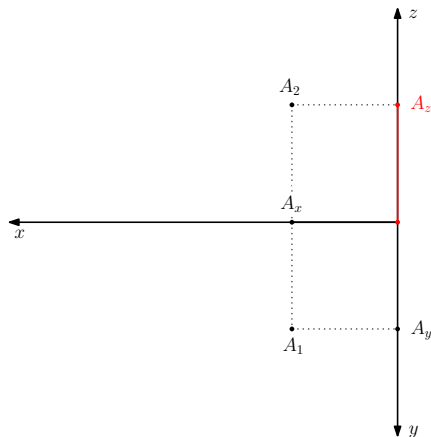
Bod



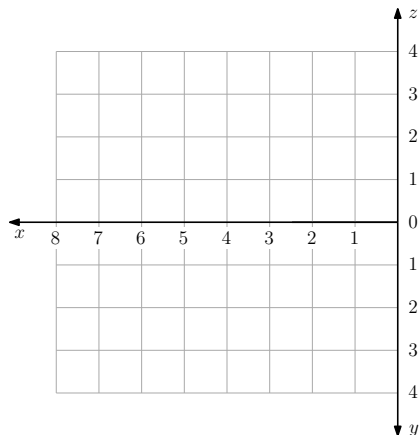
Bod



Bod



Bod

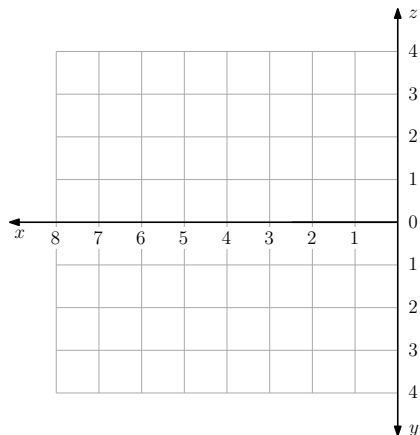


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$A = [1, 2, 3]$$

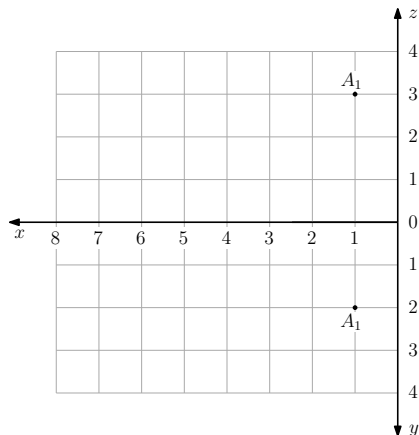


Základní úlohy Mongeovho promítání

└ Bod, přímka a rovina

└ Bod

$$A = [1, 2, 3]$$

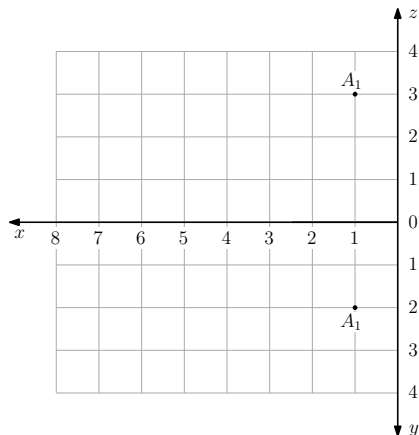


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$B = [5, 4, 2]$$

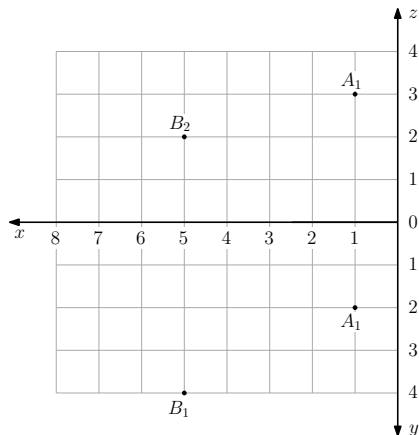


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$B = [5, 4, 2]$$

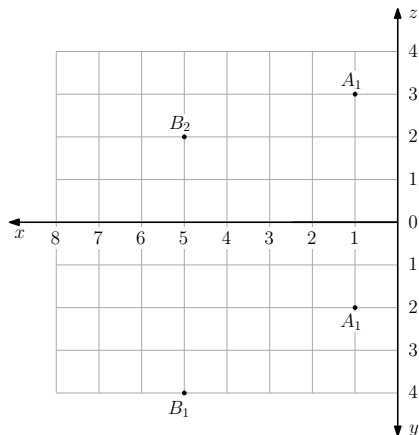


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$C = [3, -2, 3]$$

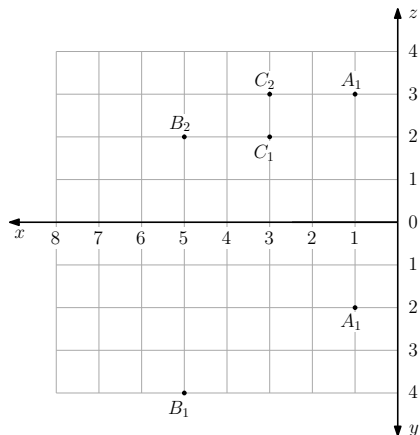


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$C = [3, -2, 3]$$

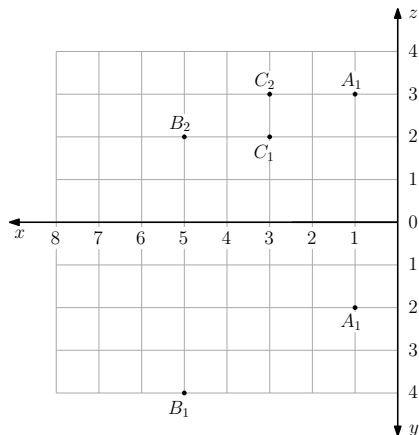


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$D = [7, -1, -2]$$

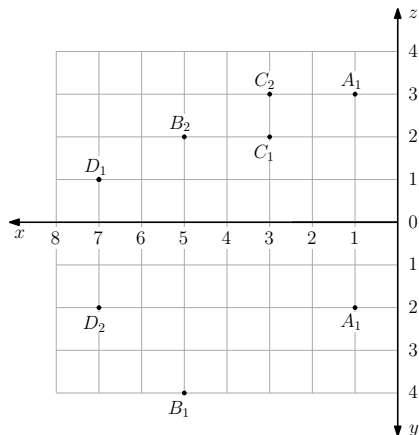


Základní úlohy Mongeovho promítání

└ Bod, priamka a rovina

└ Bod

$$D = [7, -1, -2]$$

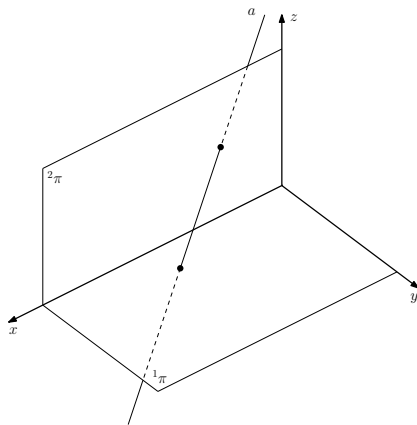


2.

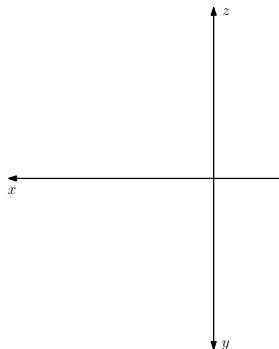
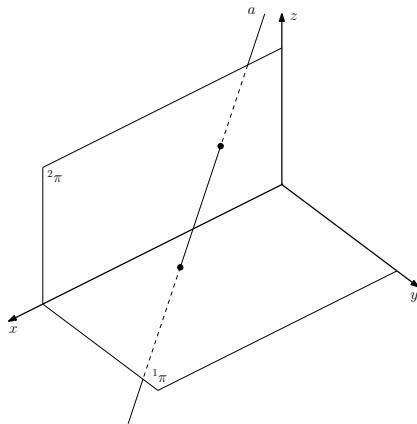
PRIAMKA V MP

STOPNÍKY

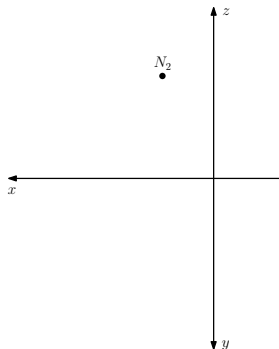
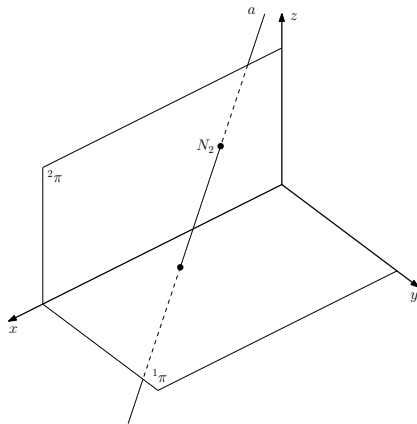
Priamka



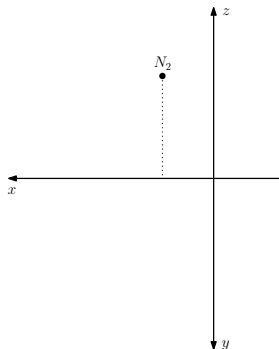
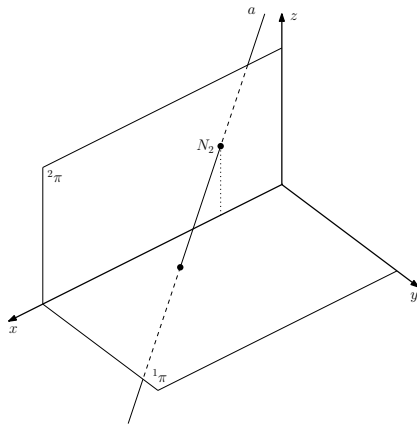
Priamka



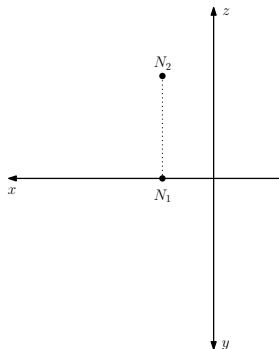
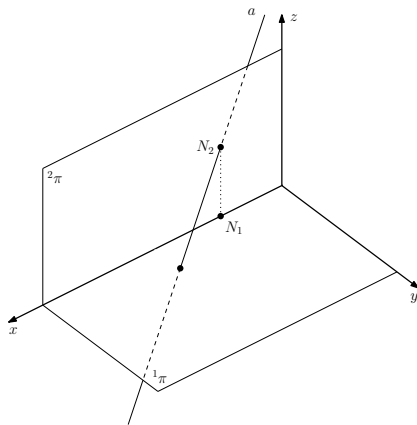
Nárysny stopník



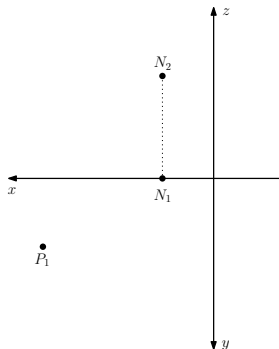
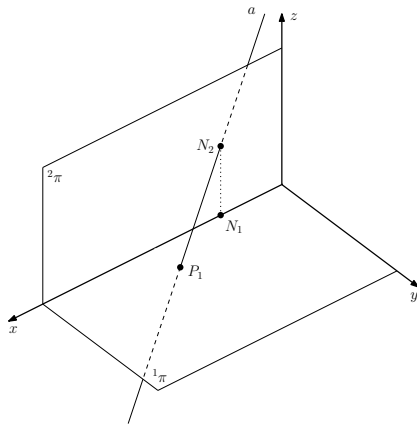
Nárysny stopník



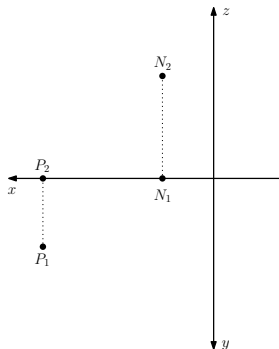
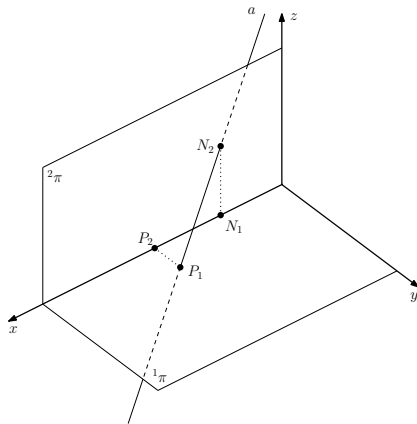
Nárysný stopník (pôdorys)



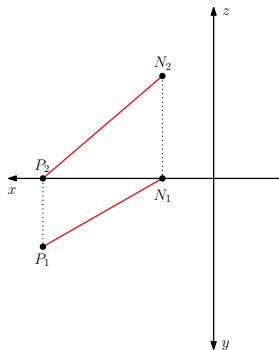
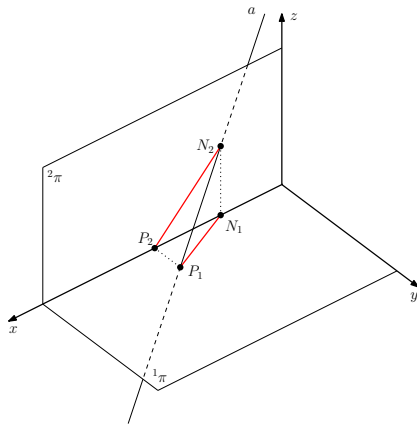
Pôdorysný stopník



Pôdorysný stopník (nárys)

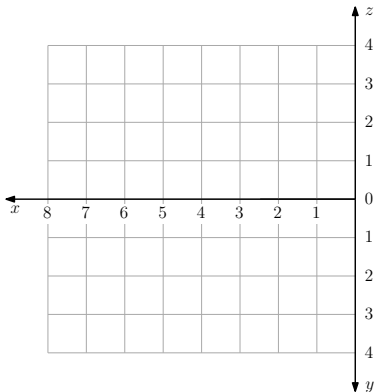


Priamka v MPs



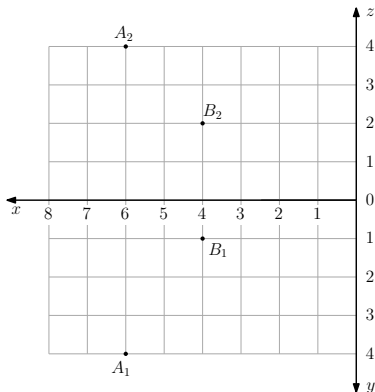
Príklad

Nájděte stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



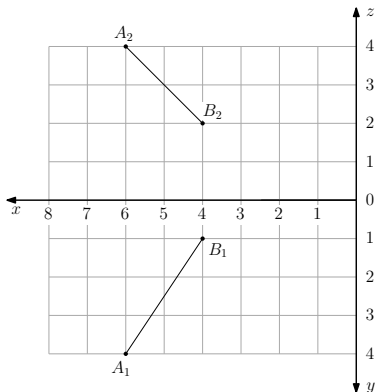
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



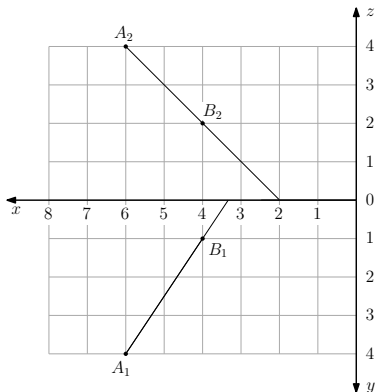
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



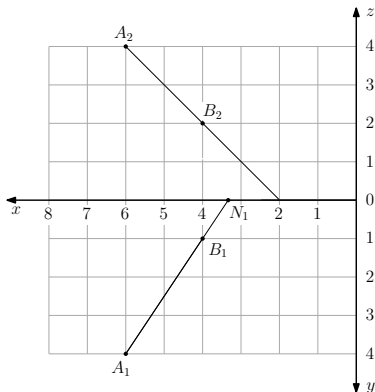
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



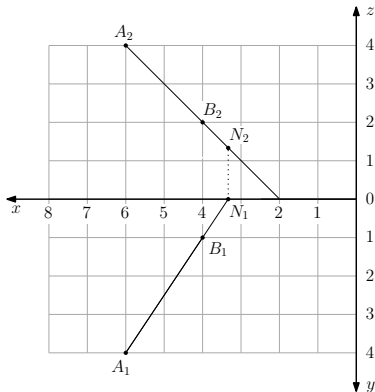
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



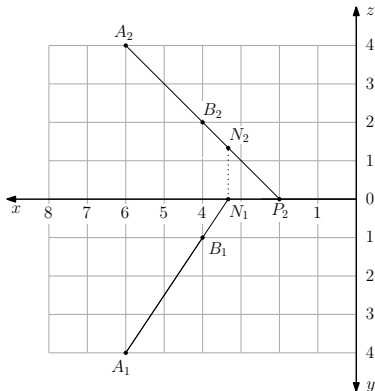
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



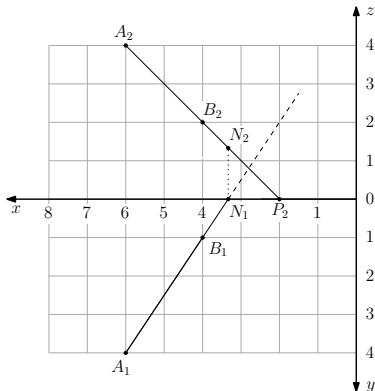
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



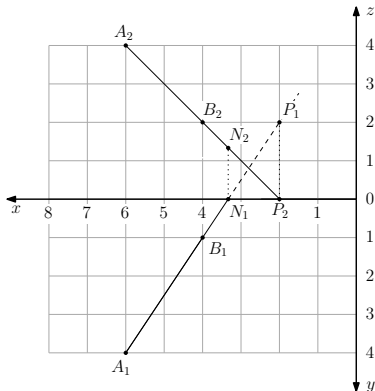
Príklad

Nájdete stopníky přímky určené body $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



Príklad

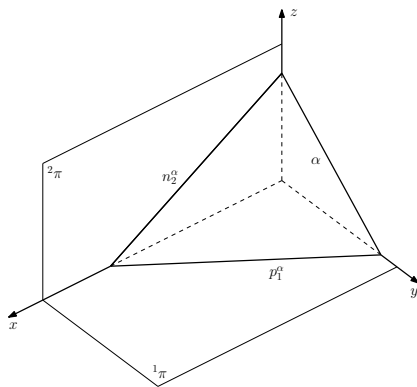
Nájdete stopníky priamky určenej bodmi $A = [6, 4, 4]$ a $B = [4, 1, 2]$.



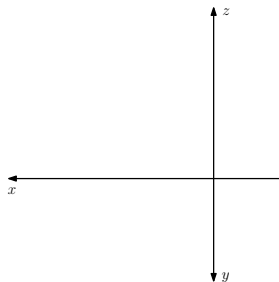
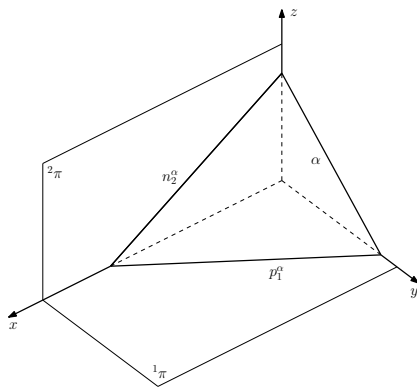
3.

ROVINA

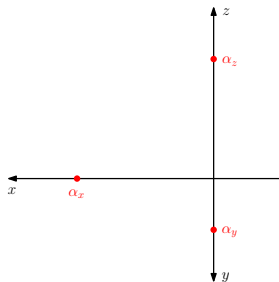
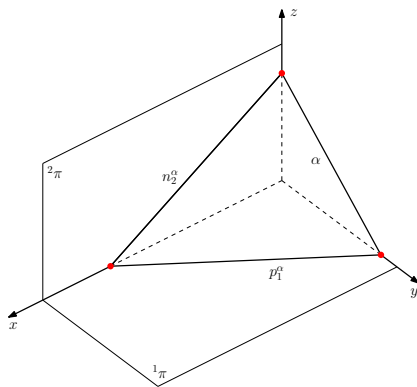
Rovina



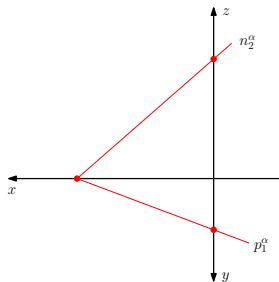
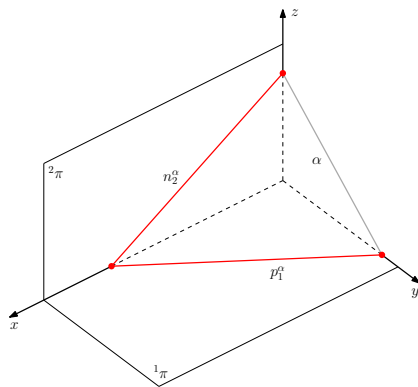
Rovina



Rovina



Rovina

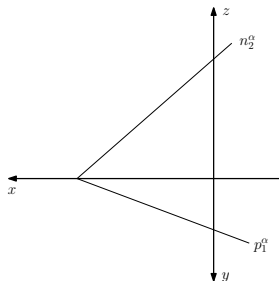
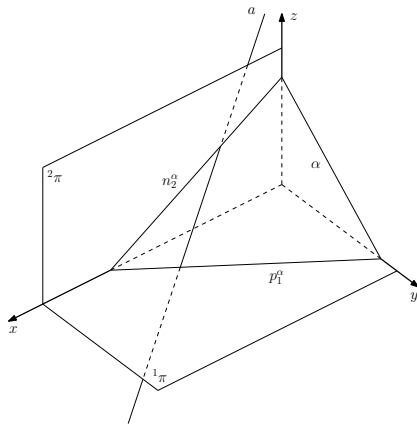


4.

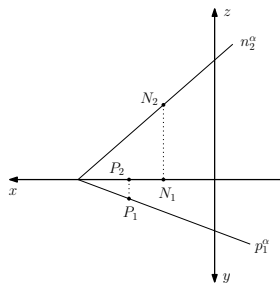
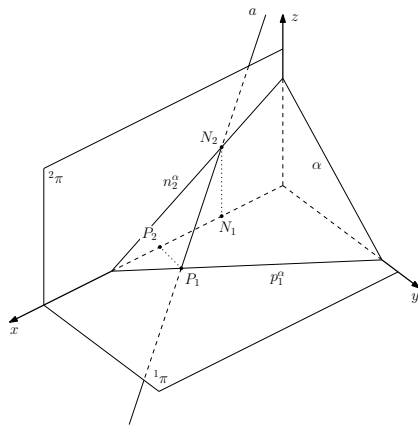
PRIAMKA A ROVINA V MP

HLAVNÉ PRIAMKY

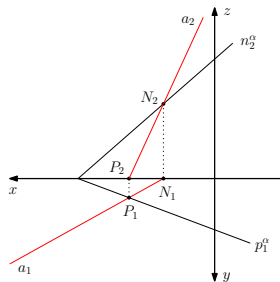
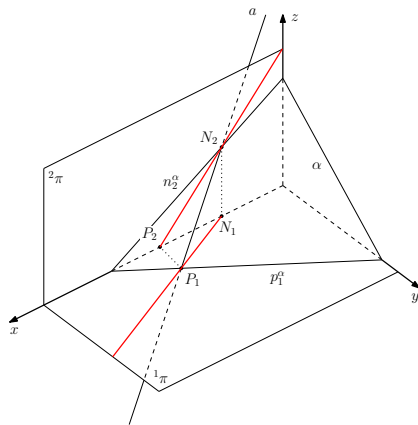
Priamka v rovine



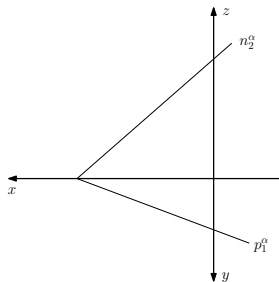
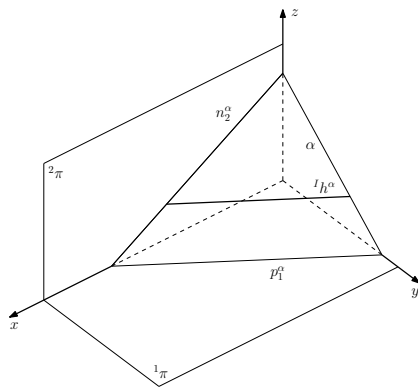
Priamka v rovine



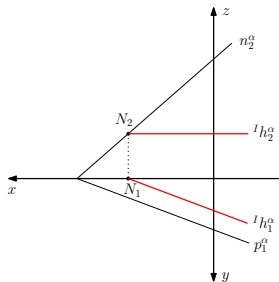
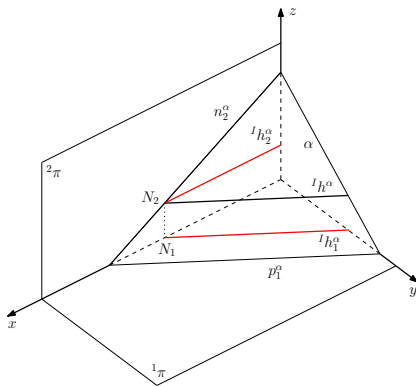
Priamka v rovine



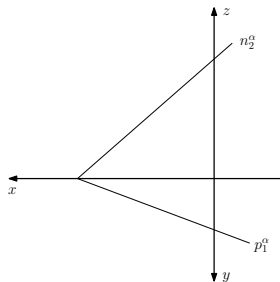
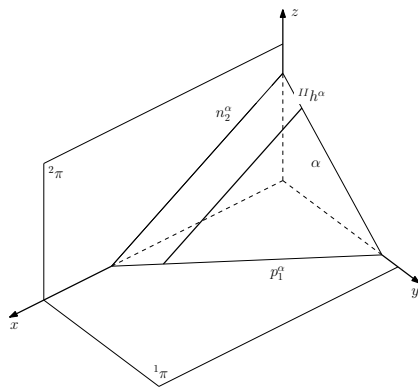
Hlavné přímky I. osnovy



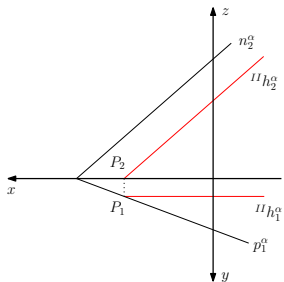
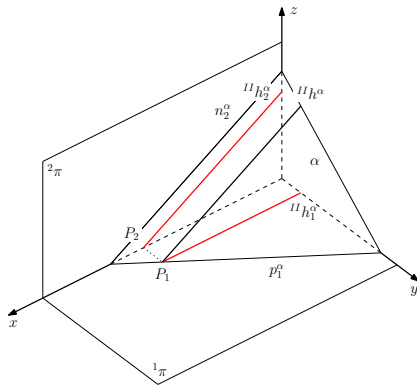
Hlavné přímky I. osnovy



Hlavné priamky II. osnovy



Hlavné přímky II. osnovy



Príklady

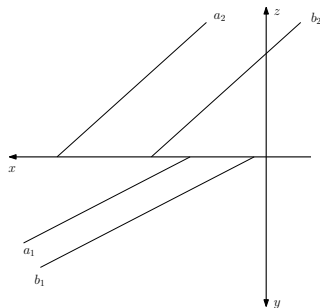
str. 36

Zostrojte nárys bodu A roviny α , ak sú dané jej stopy p^α , n^α a pôdorys A_1 bodu A .

Příklad

str. 37

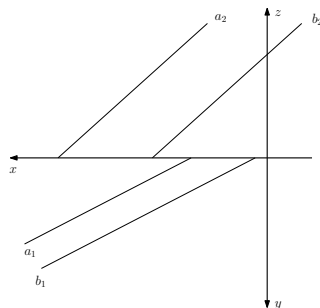
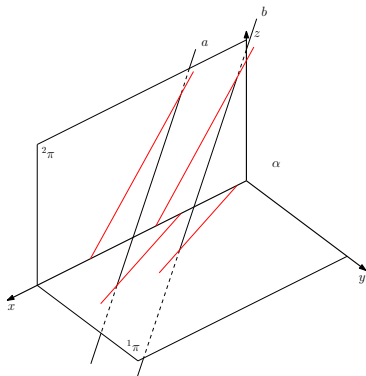
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

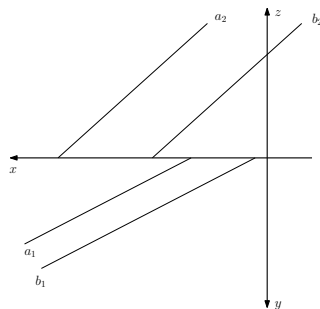
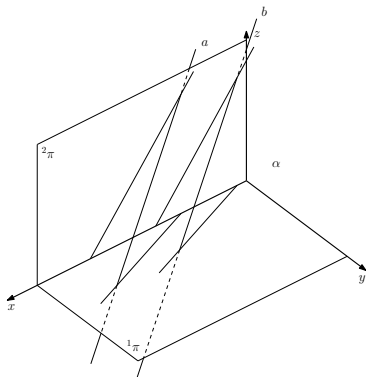
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

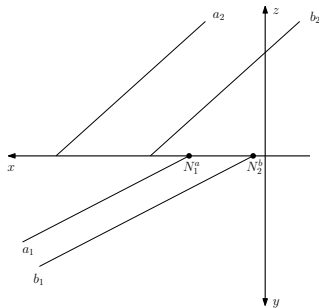
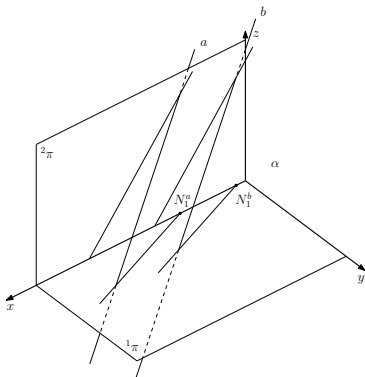
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

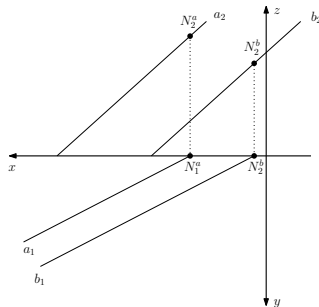
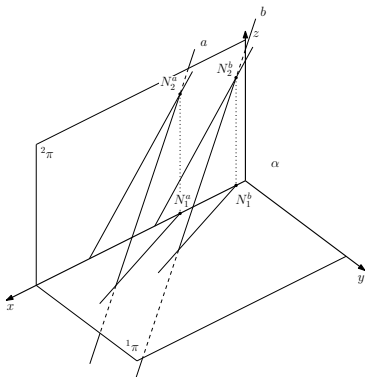
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

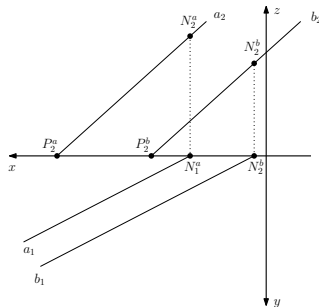
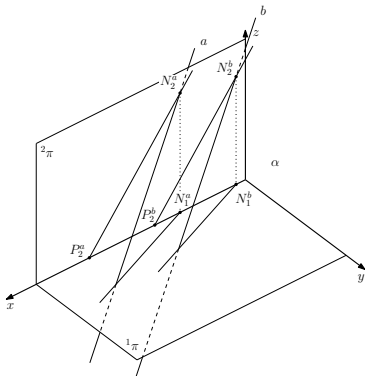
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

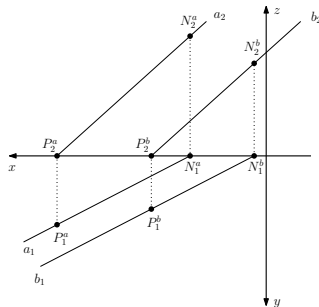
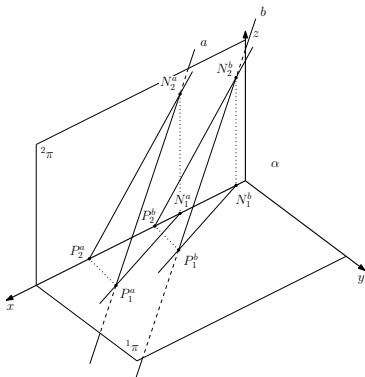
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



Příklad

str. 37

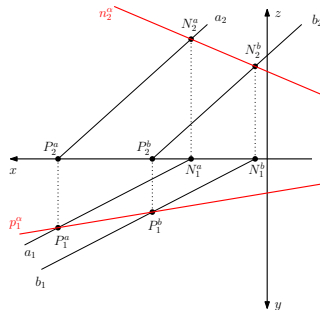
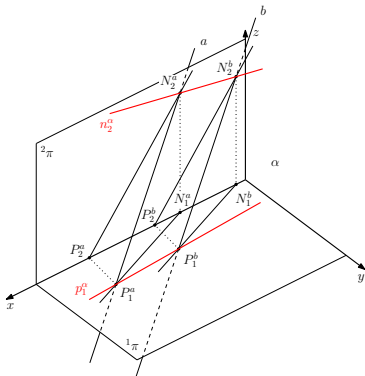
Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .



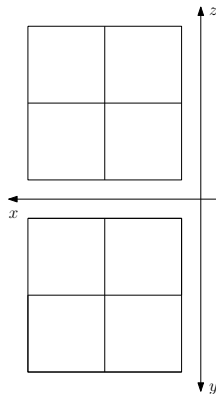
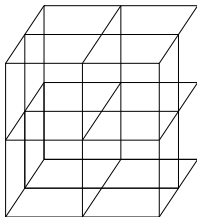
Příklad

str. 37

Zobrazte stopy roviny α určené rovnoběžnými přímkami a, b .

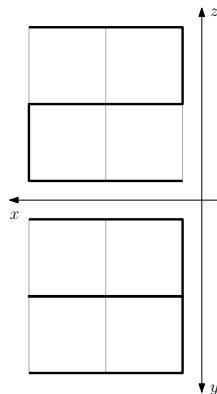
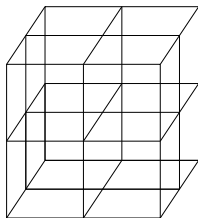


Hlavlom - Zadaní



Hlavlom - Zadaní

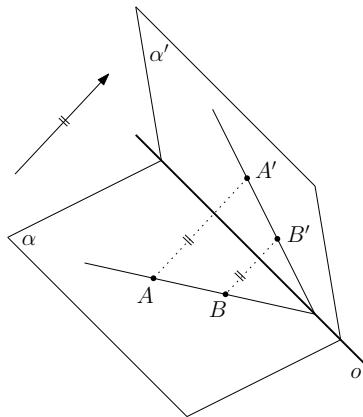
Načrtnite priestorovú lomenú čiaru jedním ťahom tak, aby v Mongeovom priemietaní vytvárala obrazec uvedený na obrázku vpravo.



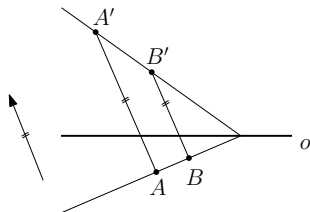
5.

OSO VÁ AFINITA

Osová afinita medzi dvoma rovinami



Osová afinita v rovine



6.

POLOHOVÉ ÚLOHY

Rozdelenie

POLOHOVÉ ÚLOHY

- ▶ daným bodom viesť k danej priamke rovnobežnú priamku
- ▶ daným bodom viesť k danej rovine rovnobežnú rovinu
- ▶ zostrojiť priesečník daných dvoch rovín
- ▶ zostrojiť priesečník danej priamky s danou rovinou

METRICKÉ ÚLOHY

- ▶ daným bodom viesť k danej rovine kolmú priamku
- ▶ daným bodom viesť k danej priamke kolmú rovinu
- ▶ sklápanie roviny
- ▶ otáčanie roviny

Polohové úlohy

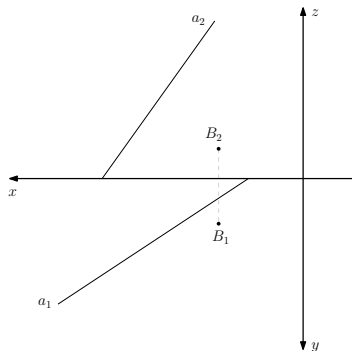
38/A1

Daným bodom B veďte priamku b rovnobežnú s danou priamkou a .

Polohové úlohy

38/A1

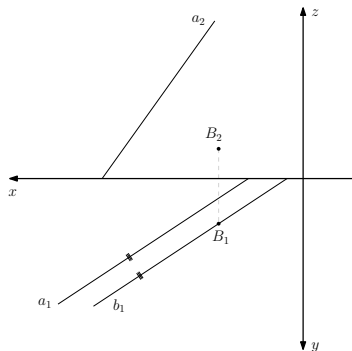
Daným bodom B veďte priamku b rovnobežnú s danou priamkou a .



Polohové úlohy

38/A1

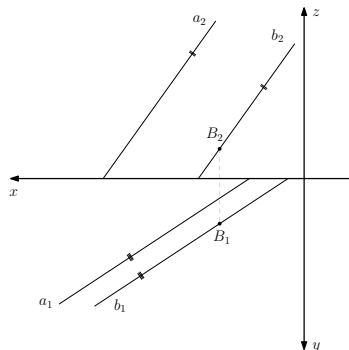
Daným bodom B veďte priamku b rovnobežnú s danou priamkou a .



Polohové úlohy

38/A1

Daným bodom B veďte priamku b rovnobežnú s danou priamkou a .

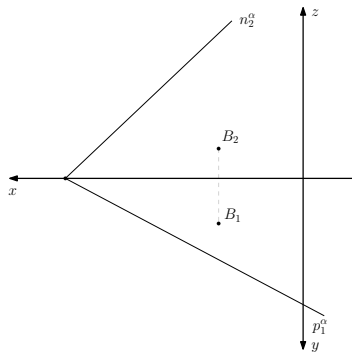


38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .

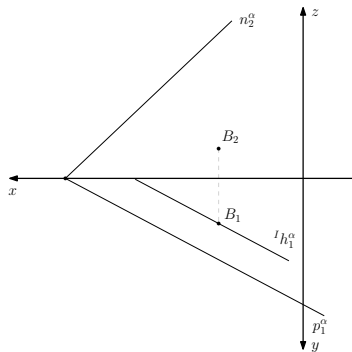
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



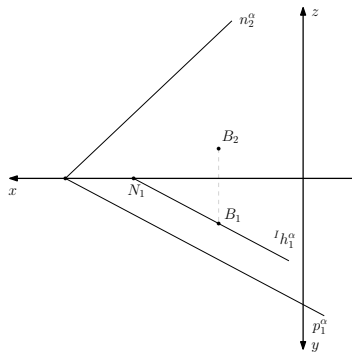
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



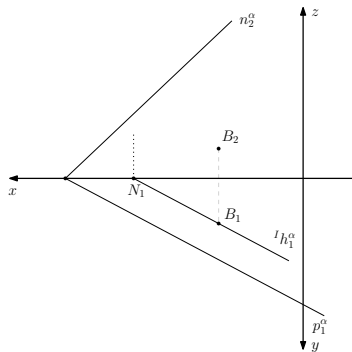
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



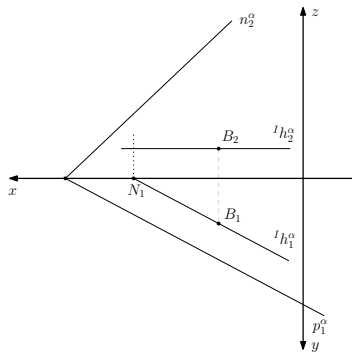
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



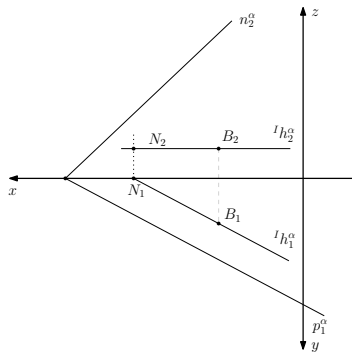
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



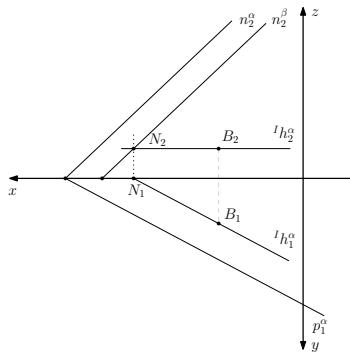
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



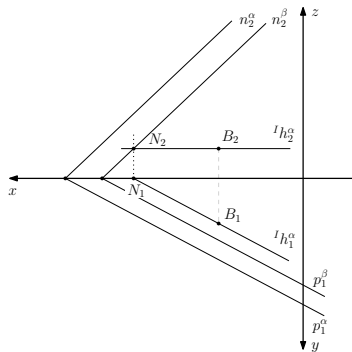
38/A2

Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



38/A2

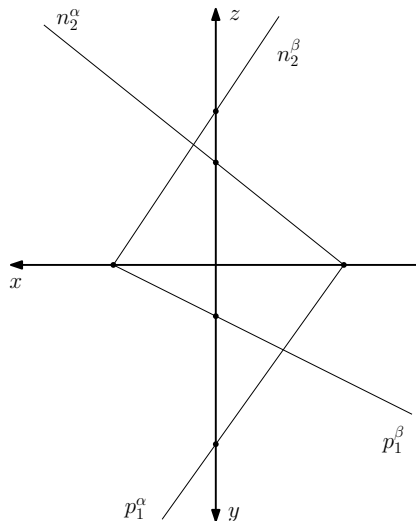
Daným bodem B veďte rovinu β rovnoběžnou s rovinou danou rovinou α .



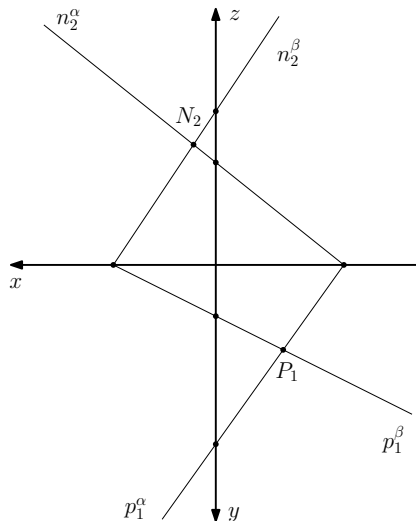
49/14a

Sestrojte průsečnici rovin $\alpha(-50, 70, 40)$, $\beta(40, 20, 60)$.

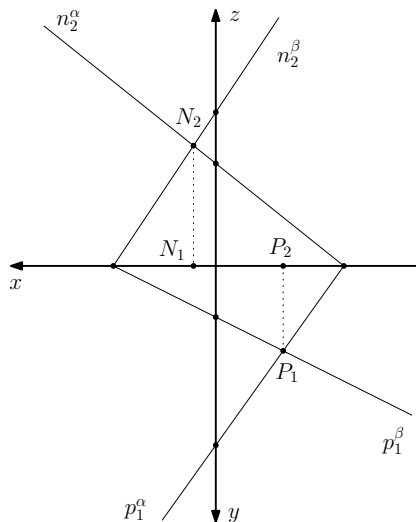
Polohové úlohy



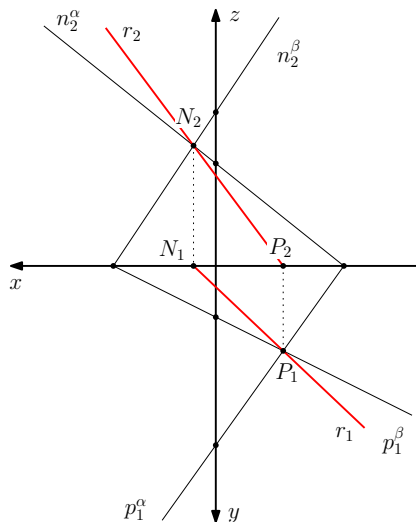
Polohové úlohy



Polohové úlohy



Polohové úlohy



Polohové úlohy

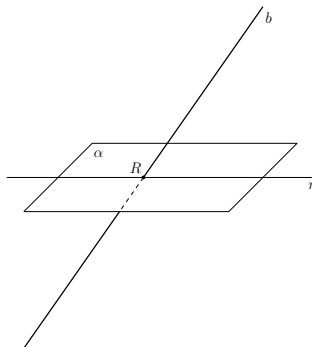
49/16a

Sestrojte průsečníky přímky $b = AB$, kde $A = [0, 30, 40]$ a $B = [-50, 10, 30]$ s rovinou $\alpha(30, 20, 20)$.

Polohové úlohy

49/16a

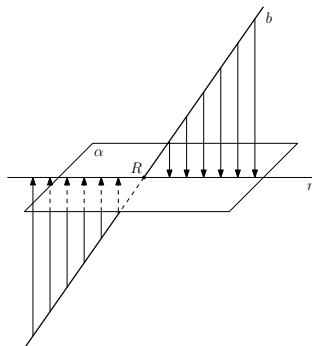
Sestrojte průsečníky přímky $b = AB$, kde $A = [0, 30, 40]$ a $B = [-50, 10, 30]$ s rovinou $\alpha(30, 20, 20)$.



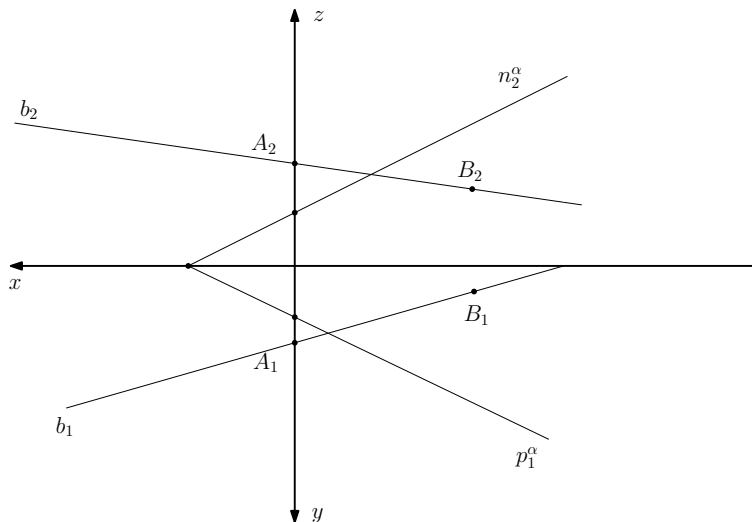
Polohové úlohy

49/16a

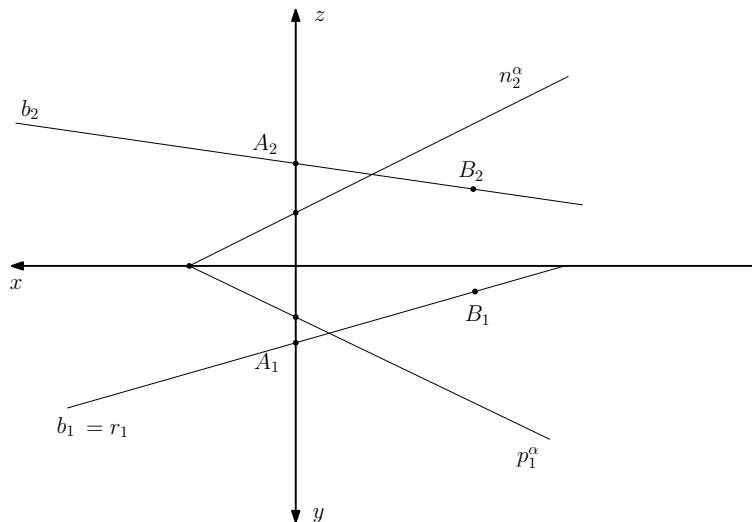
Sestrojte průsečníky přímky $b = AB$, kde $A = [0, 30, 40]$ a $B = [-50, 10, 30]$ s rovinou $\alpha(30, 20, 20)$.



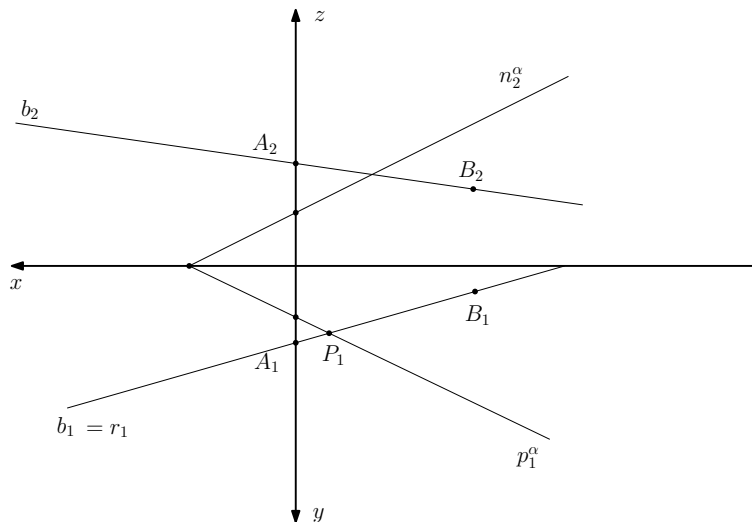
Polohové úlohy



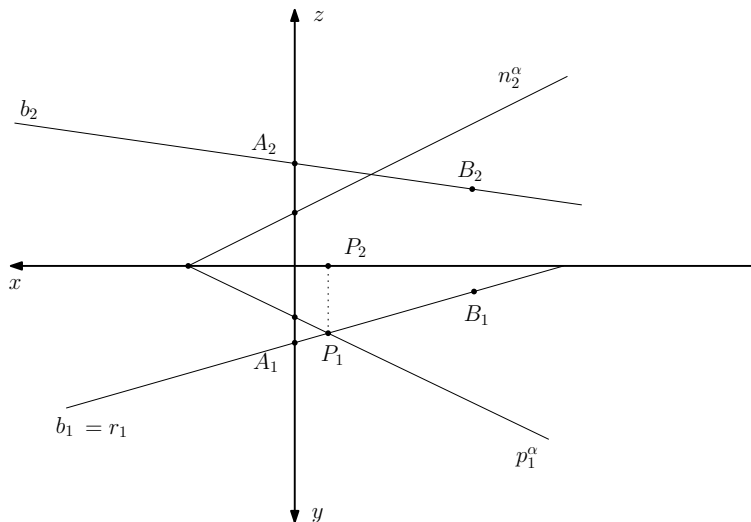
Polohové úlohy



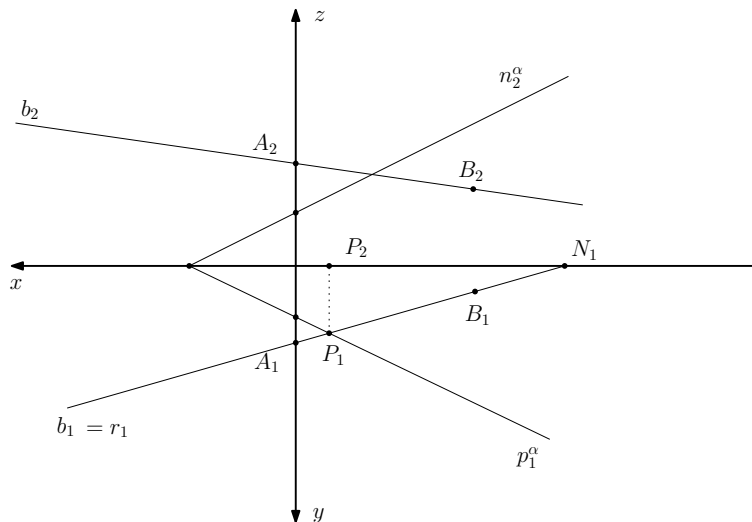
Polohové úlohy



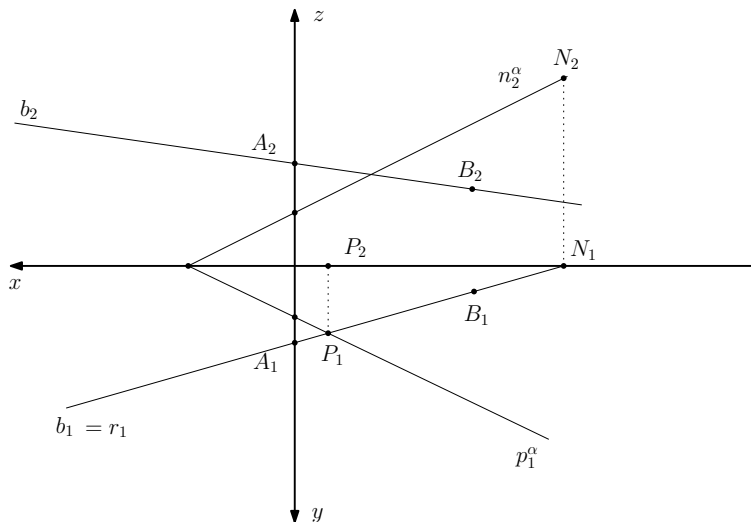
Polohové úlohy



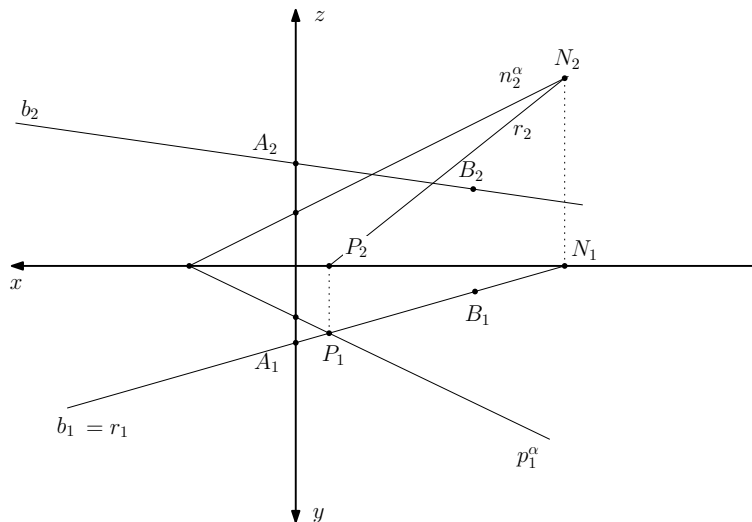
Polohové úlohy



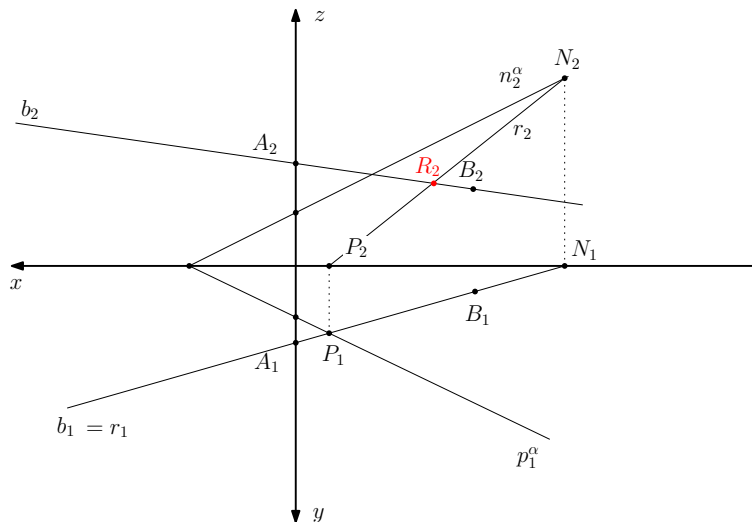
Polohové úlohy



Polohové úlohy



Polohové úlohy



7.

METRICKÉ ÚLOHY

Rozdelenie

POLOHOVÉ ÚLOHY

- ▶ daným bodom viesť k danej priamke rovnobežnú priamku
- ▶ daným bodom viesť k danej rovine rovnobežnú rovinu
- ▶ zostrojiť priesečník daných dvoch rovín
- ▶ zostrojiť priesečník danej priamky s danou rovinou

METRICKÉ ÚLOHY

- ▶ daným bodom viesť k danej rovine kolmú priamku
- ▶ daným bodom viesť k danej priamke kolmú rovinu
- ▶ sklápanie roviny
- ▶ otáčanie roviny

Metrické úlohy

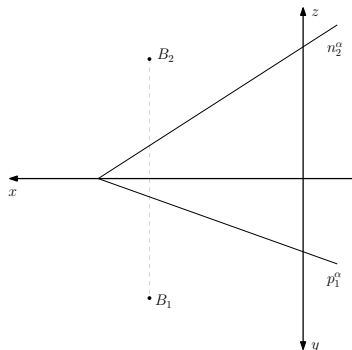
40/B1

Daným bodom B veďte priamku k kolmú k danej rovine α .

Metrické úlohy

40/B1

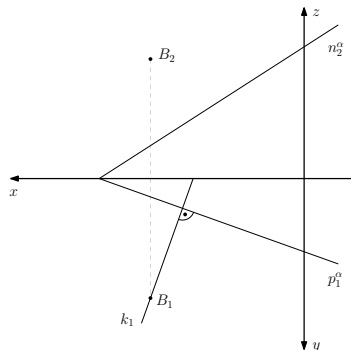
Daným bodom B veďte priamku k kolmú k danej rovine α .



Metrické úlohy

40/B1

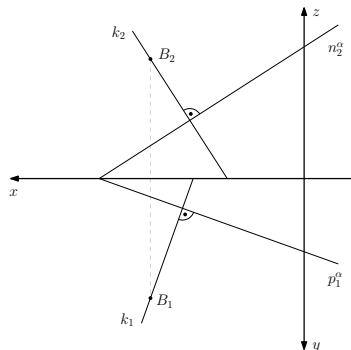
Daným bodom B veďte priamku k kolmú k danej rovine α .



Metrické úlohy

40/B1

Daným bodom B veďte priamku k kolmú k danej rovine α .



Metrické úlohy

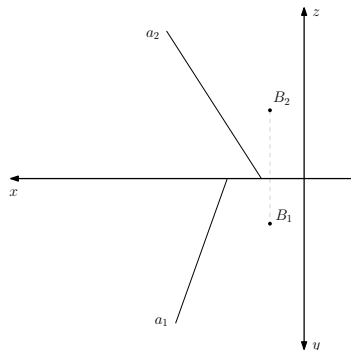
40/B2

Daným bodom B veďte k danej priamke a kolmú rovinu α .

Metrické úlohy

40/B2

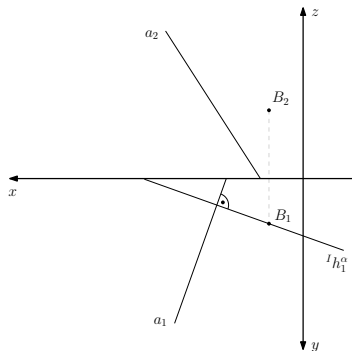
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

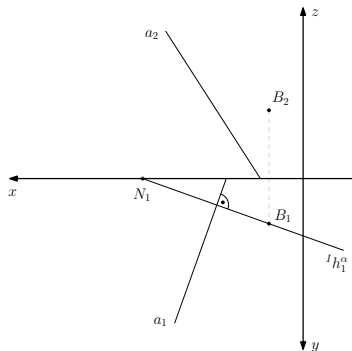
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

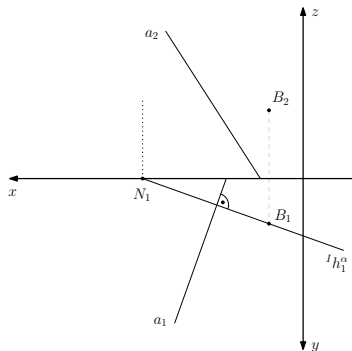
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

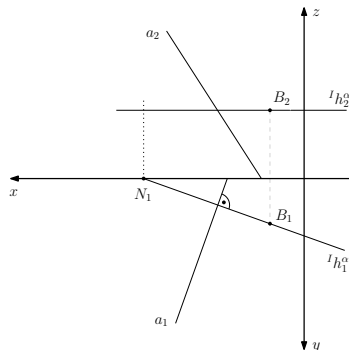
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

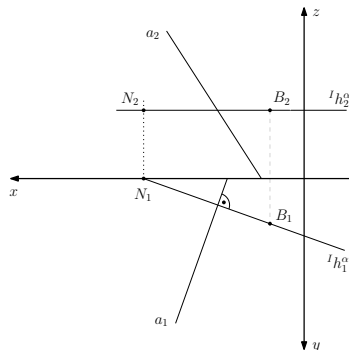
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

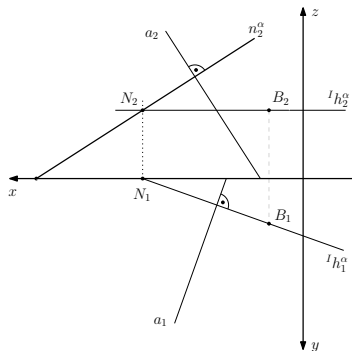
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

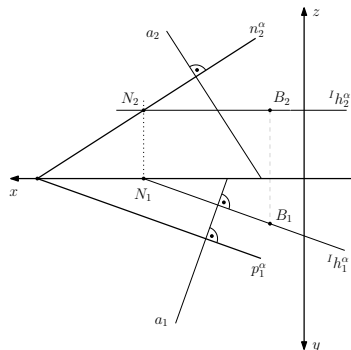
Daným bodom B veďte k danej priamke a kolmú rovinu α .



Metrické úlohy

40/B2

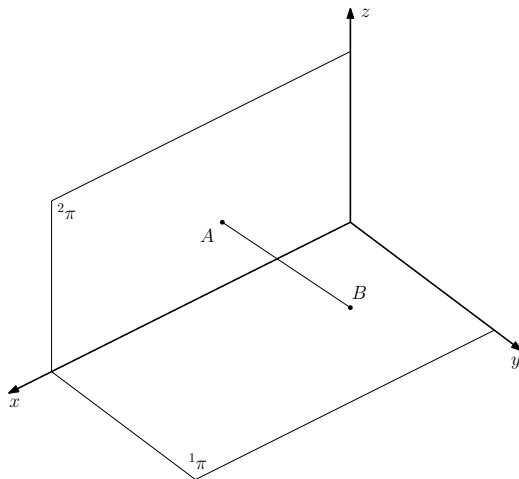
Daným bodom B veďte k danej priamke a kolmú rovinu α .



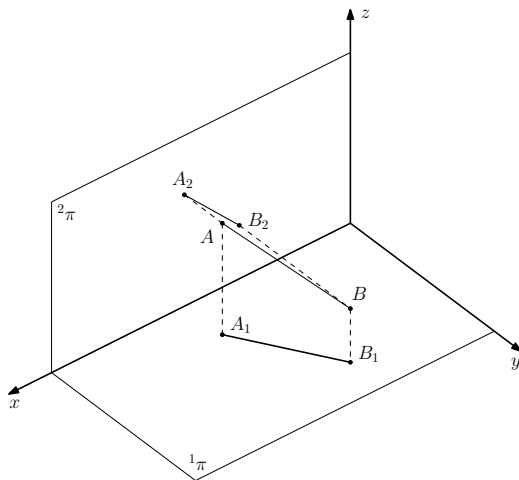
8.

SKLÁPANIE ROVINY

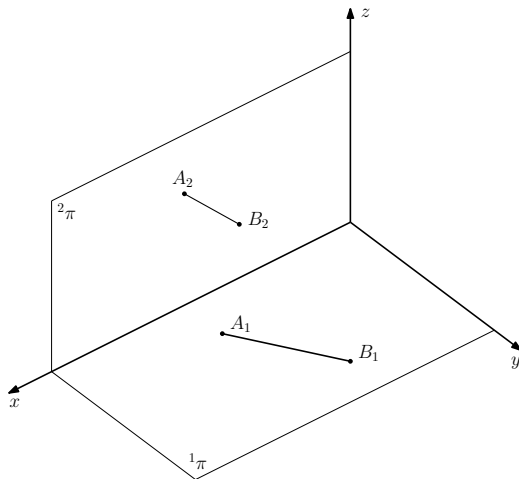
Sklápanie roviny



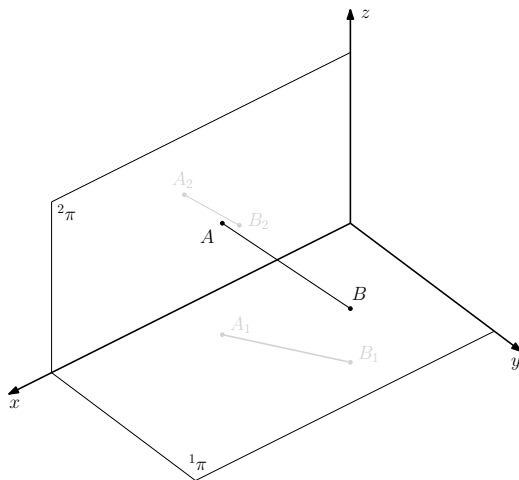
Sklápanie roviny



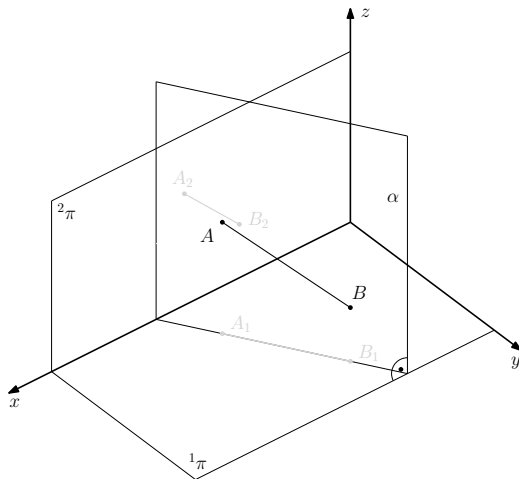
Sklápanie roviny



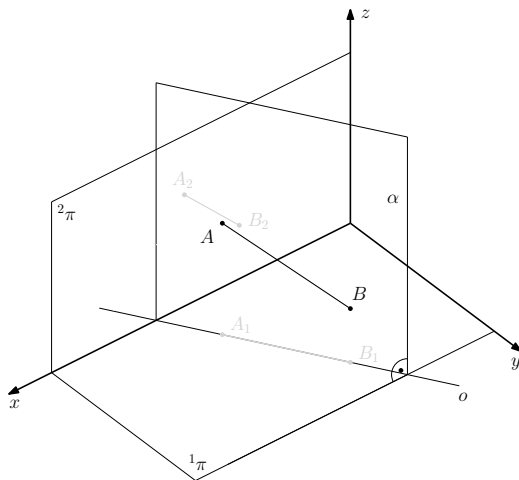
Sklápanie roviny



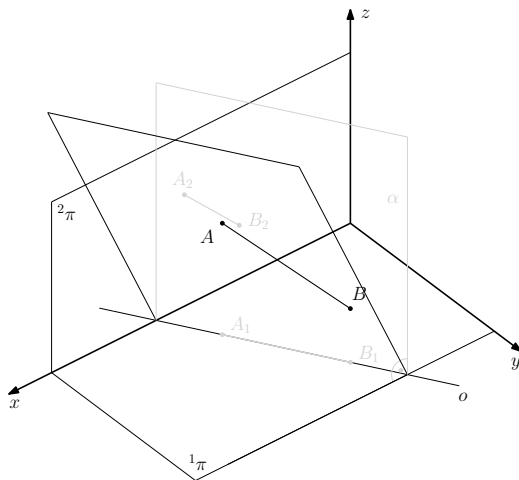
Sklápanie roviny



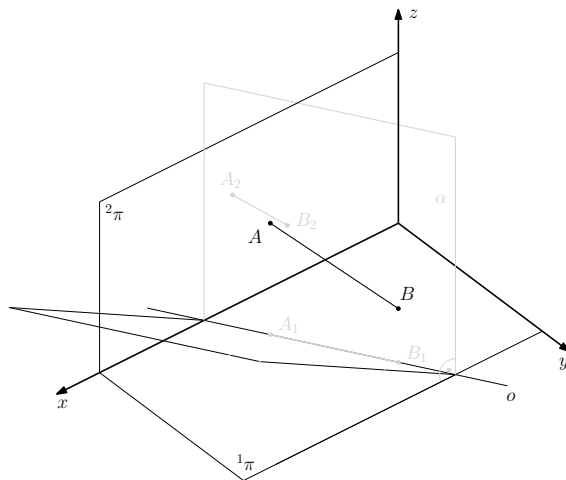
Sklápanie roviny



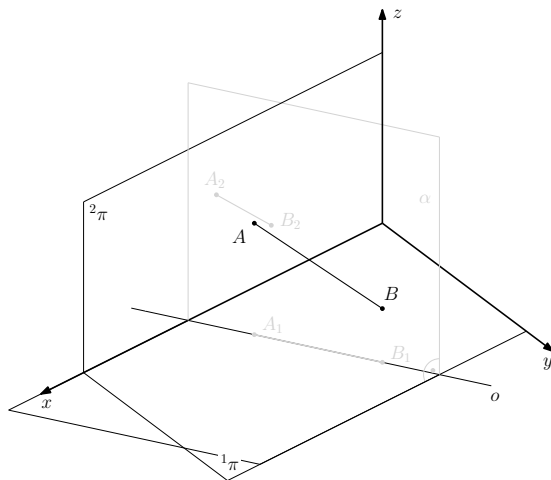
Sklápanie roviny



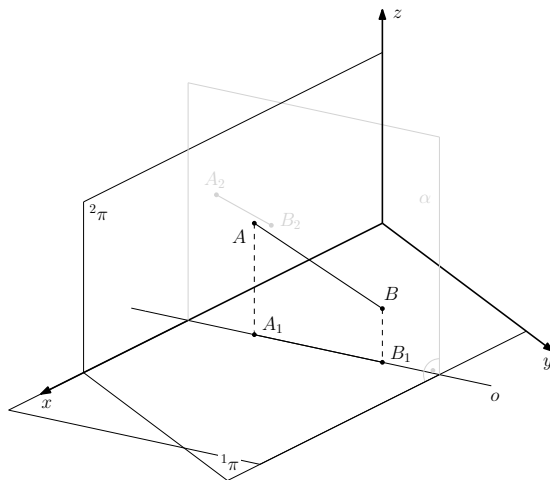
Sklápanie roviny



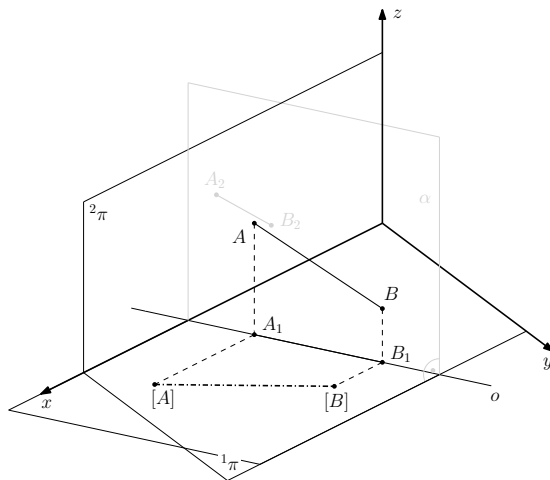
Sklápanie roviny



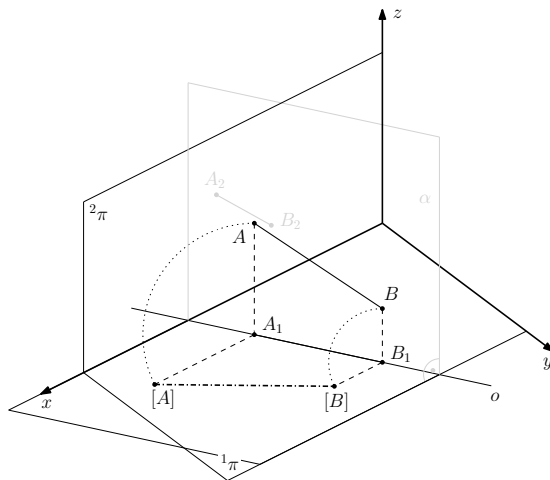
Sklápanie roviny



Sklápanie roviny



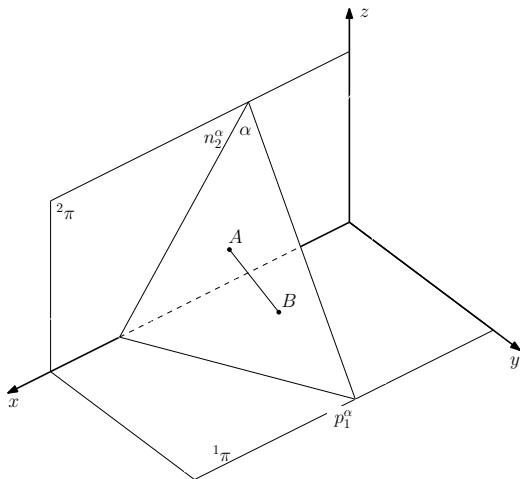
Sklápanie roviny



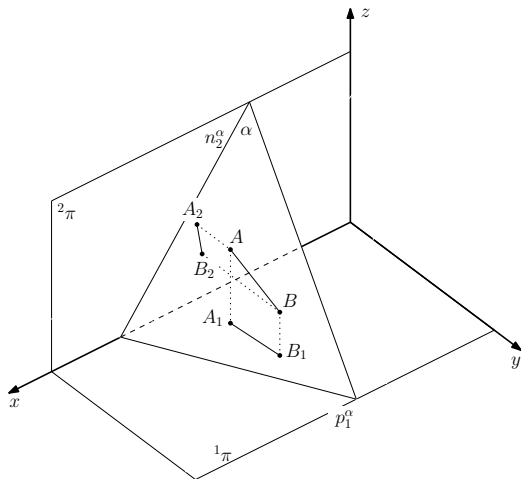
9.

OTÁČANIE ROVINY

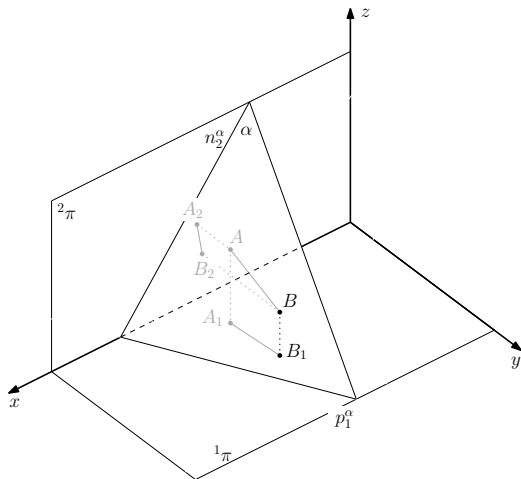
Otáčanie roviny



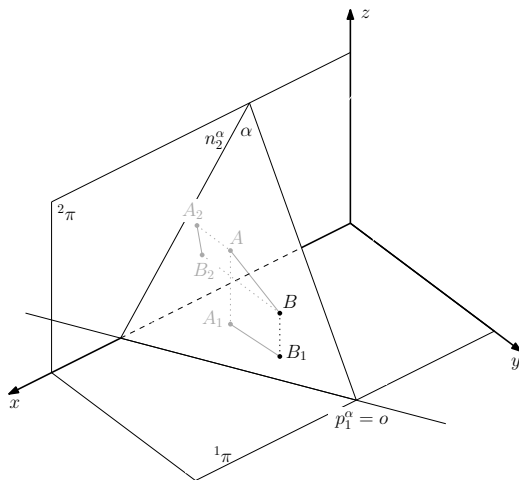
Otáčanie roviny



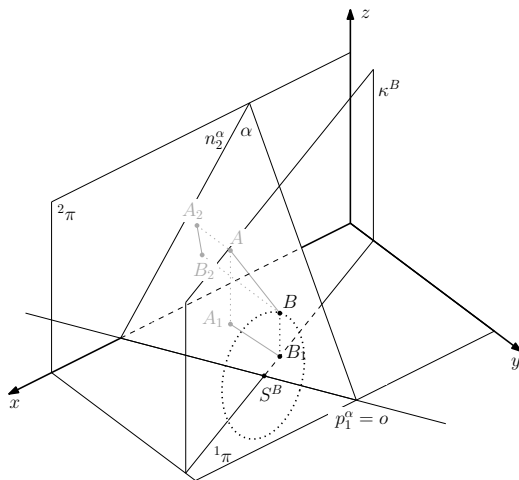
Otáčanie roviny



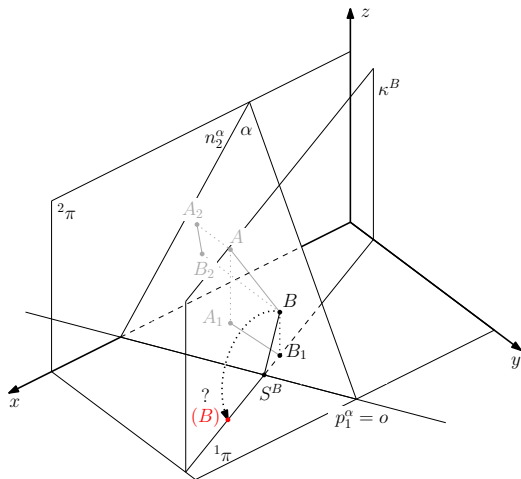
Otáčanie roviny



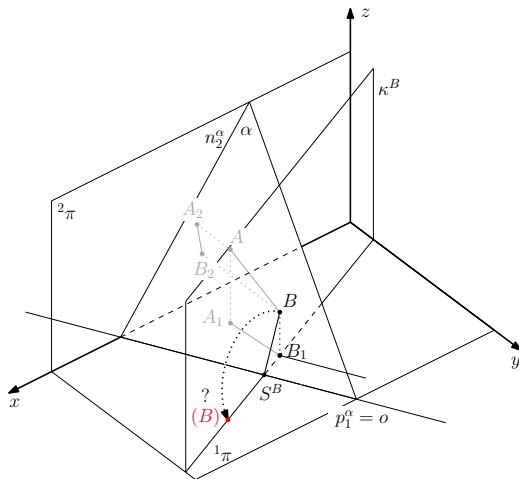
Otáčanie roviny



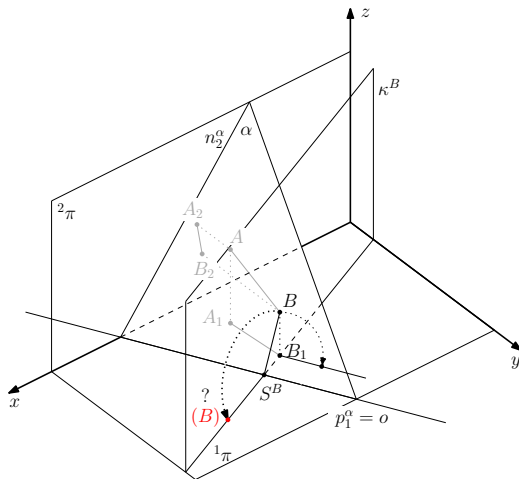
Otáčanie roviny



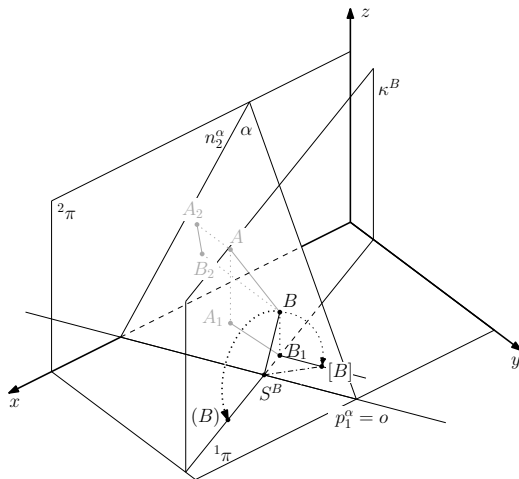
Otáčanie roviny



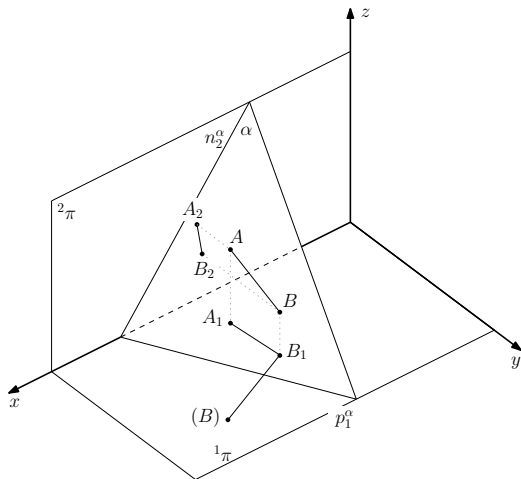
Otáčanie roviny



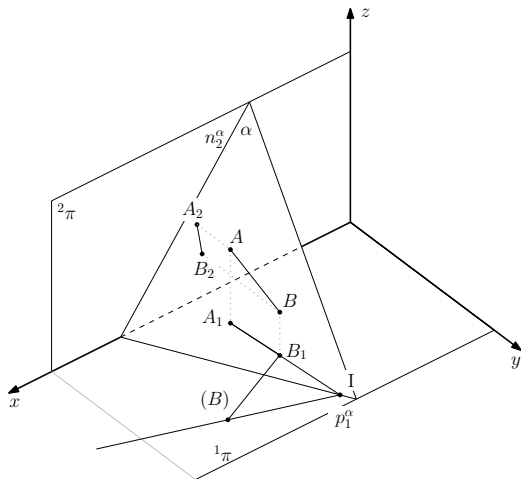
Otáčanie roviny



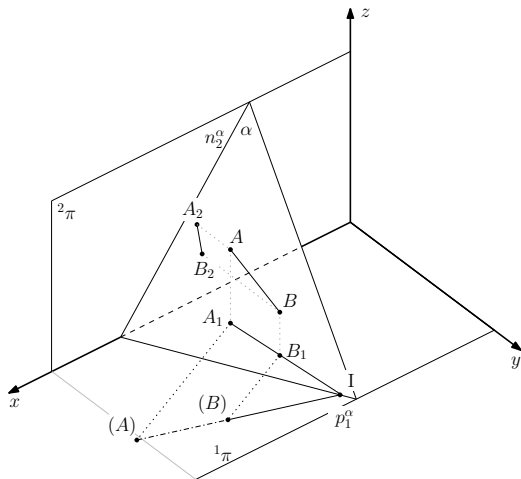
Otáčanie roviny



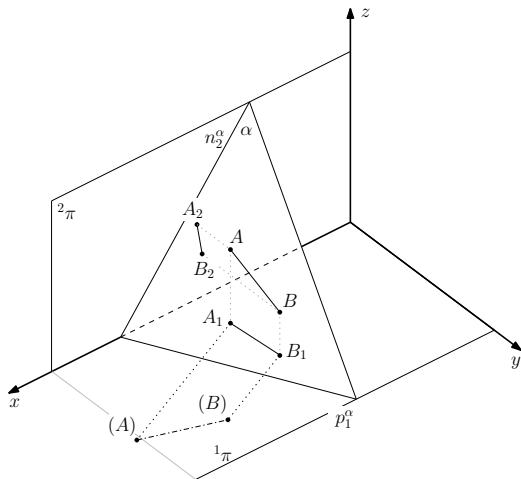
Otáčanie roviny



Otáčanie roviny



Otáčanie roviny



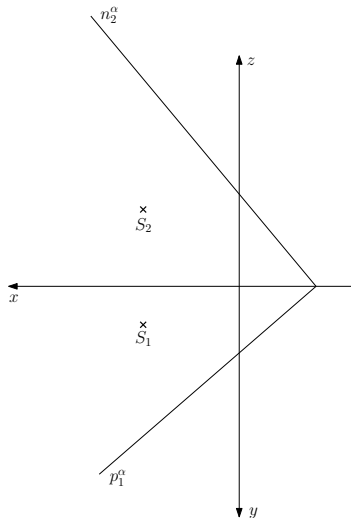
10.

TELESÁ

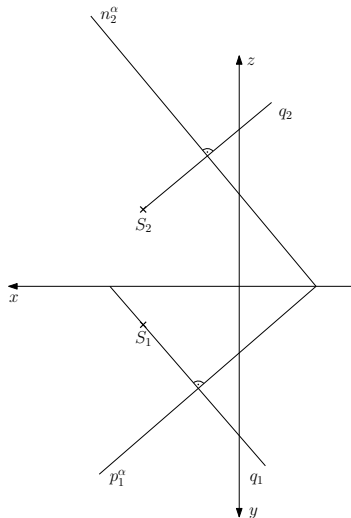
Príklad 1

Zobrazte v Mongeově promítání kulovou plochu určenou středem $S[50, 20, 40]$ a tečnou rovinou $\alpha(-40, 35, 50)$, která je zadána stopami p^α , n^α .

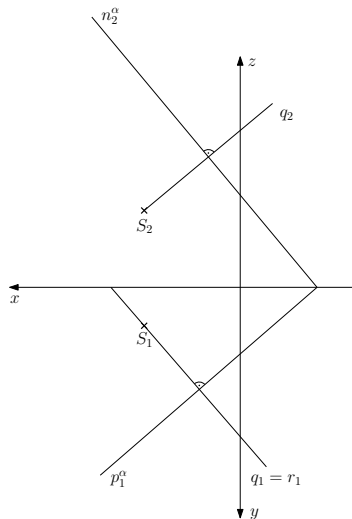
Příklad 1



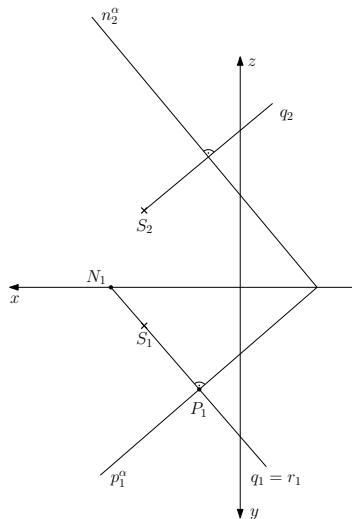
Příklad 1



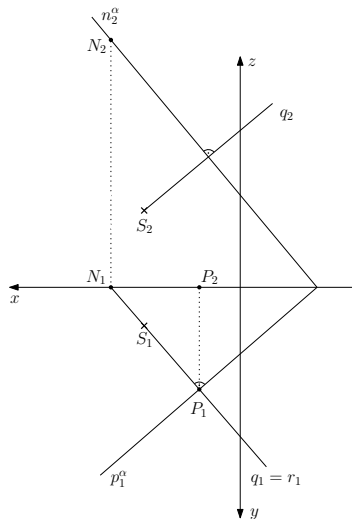
Příklad 1



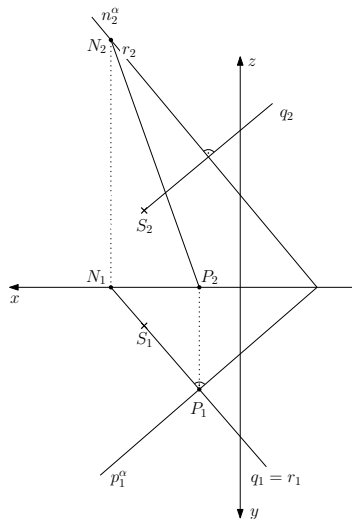
Příklad 1



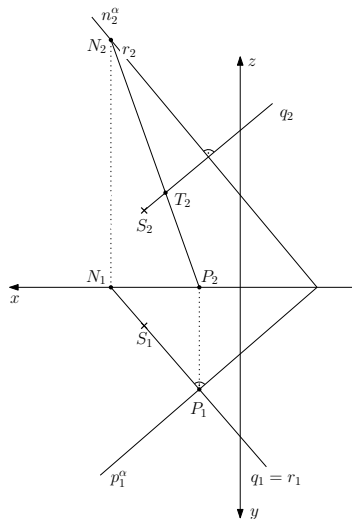
Příklad 1



Příklad 1

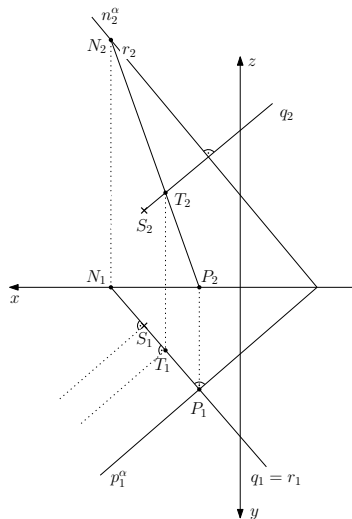


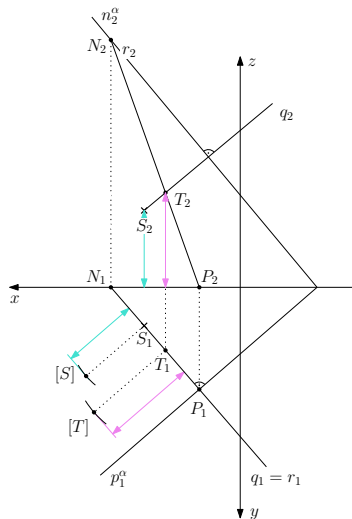
Příklad 1



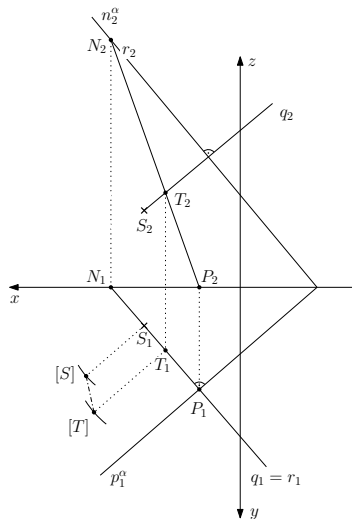
[illegible]

Příklad 1

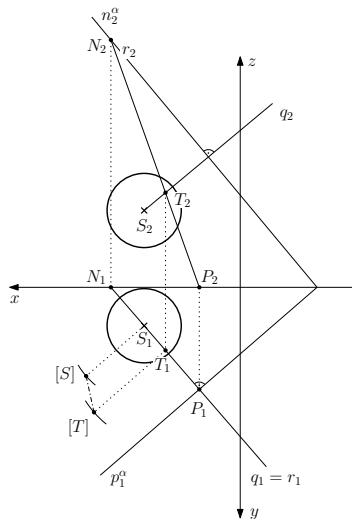




Příklad 1



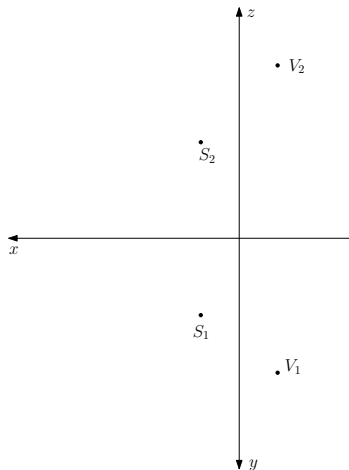
Příklad 1



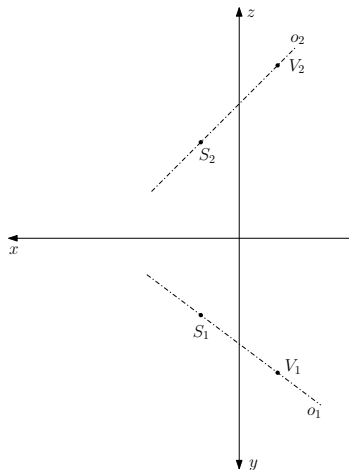
Príklad 2

V Mogeově promítání zobrazte rotační kužel, ak je daný jeho vrchol $V[-20, 90, 70]$, střed podstavy $S[20, 40, 50]$ a poloměr $r = 25$ podstavy.

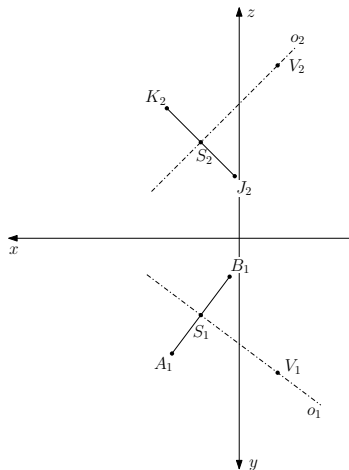
Príklad 2



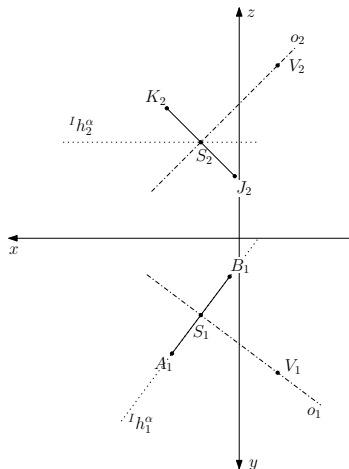
Príklad 2



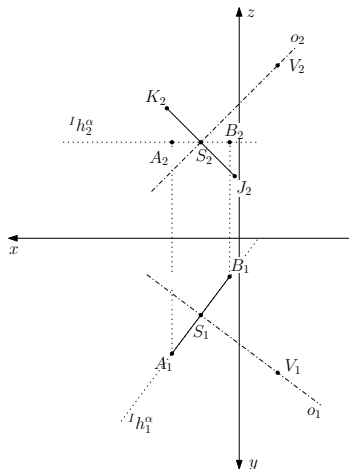
Príklad 2



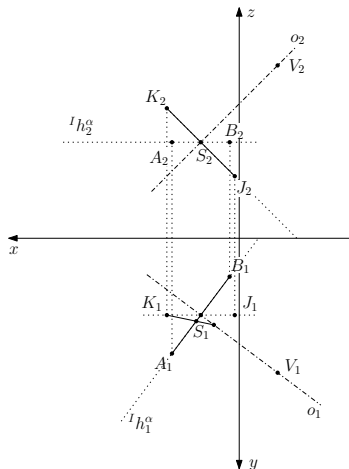
Příklad 2



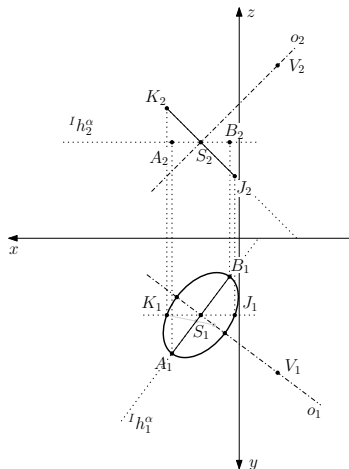
Příklad 2



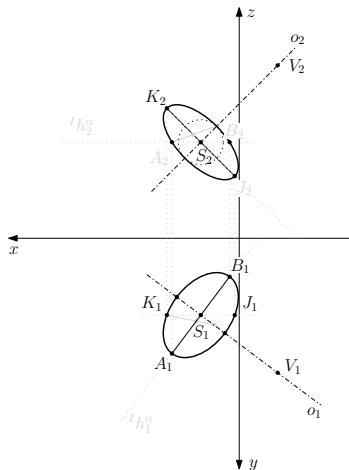
Příklad 2



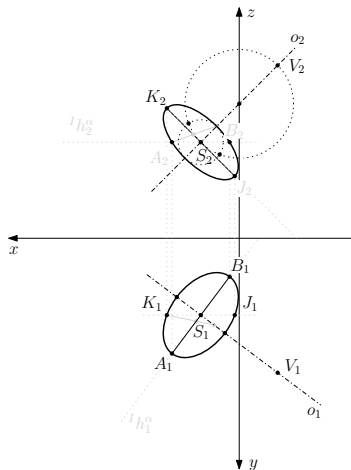
Příklad 2



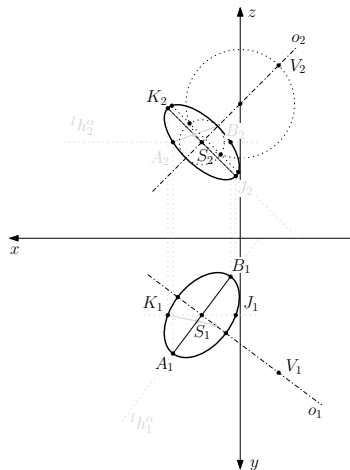
Příklad 2



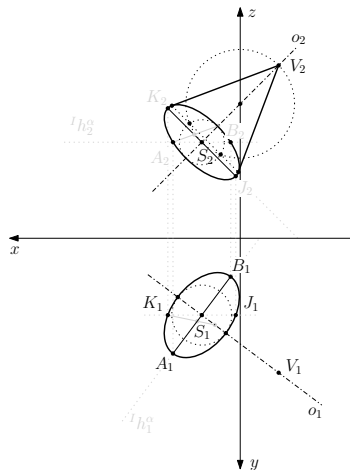
Příklad 2



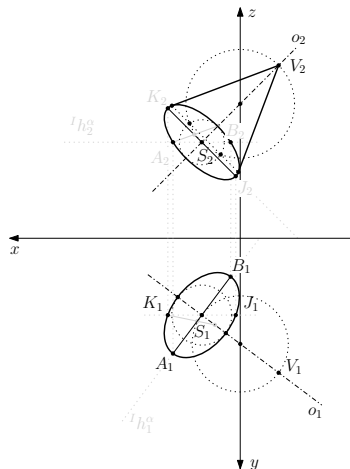
Příklad 2



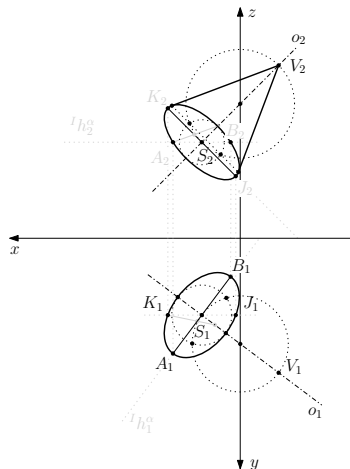
Příklad 2



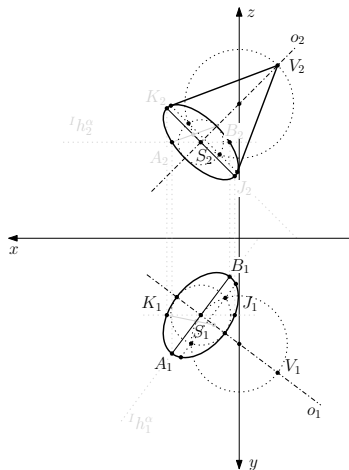
Příklad 2



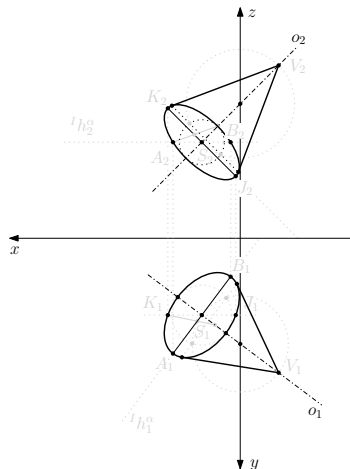
Příklad 2



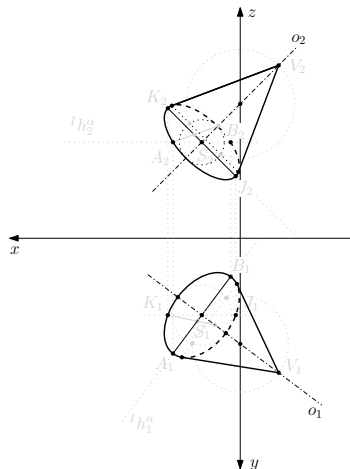
Příklad 2



Příklad 2



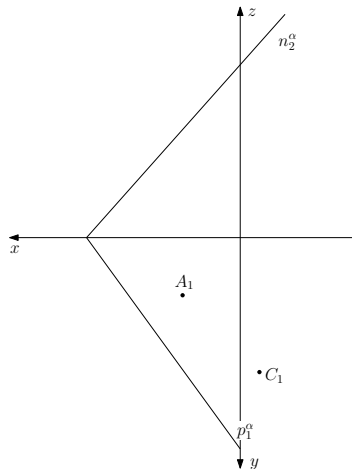
Příklad 2



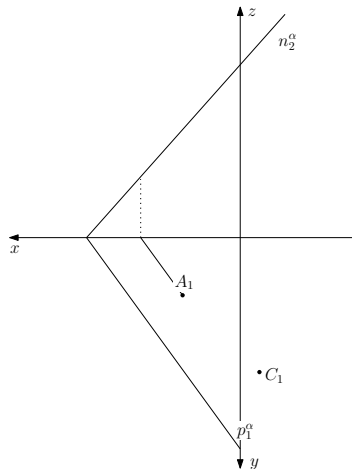
Príklad 3

V Mongeově promítání zobrazte pravidelný štvorboký hranol $ABCD\overline{A}\overline{B}\overline{C}\overline{D}$ s podstavou v $ABCD$ v rovine $\alpha(80, 110, 90)$, která je zadána stopami p^α , n^α . Hranol je zadán výškou $v = 70$ a půdorysy $A_1[30, 30, ?]$, $C_1[-10, 70, ?]$ vrcholů A , C .

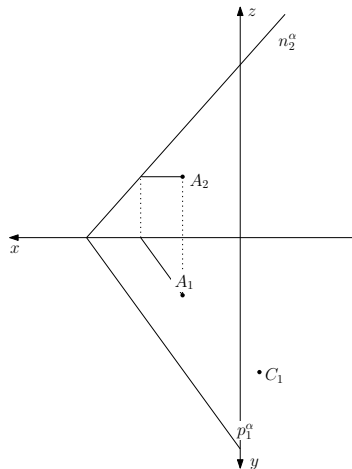
Príklad 3



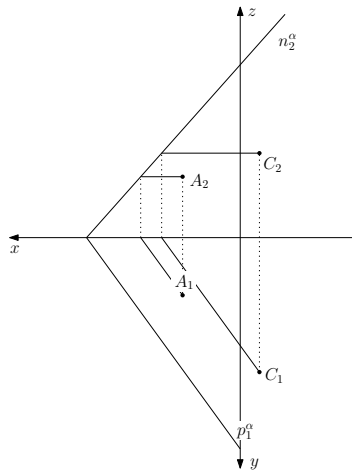
Príklad 3



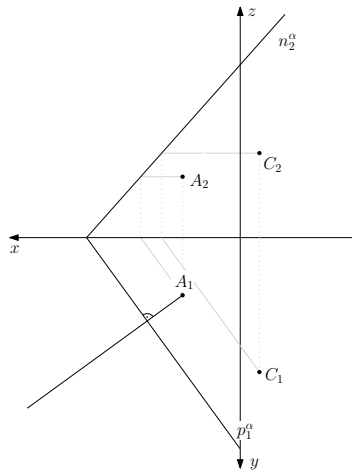
Príklad 3



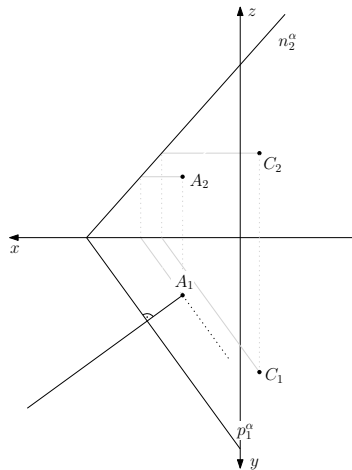
Príklad 3



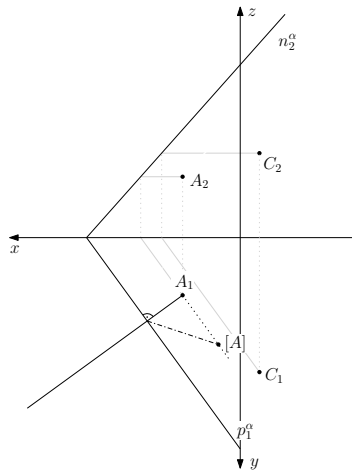
Príklad 3



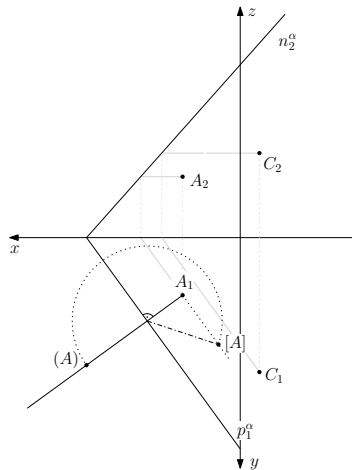
Příklad 3



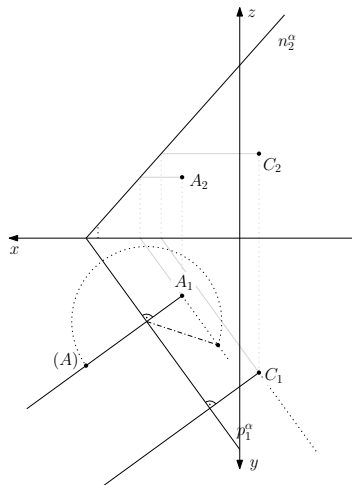
Příklad 3

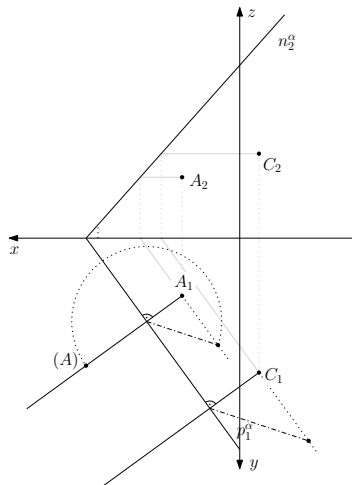


Příklad 3

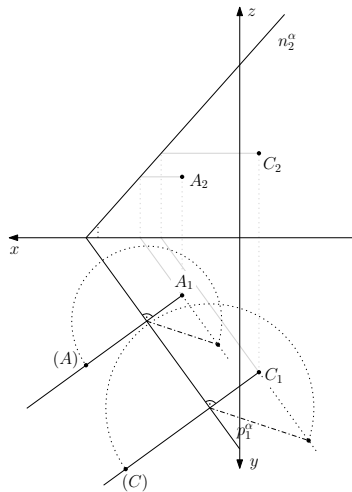


Příklad 3

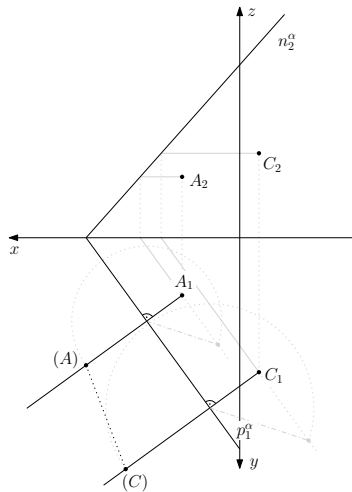




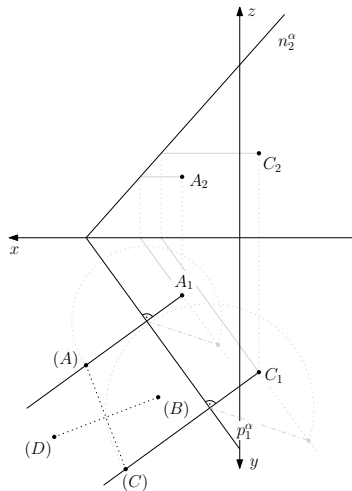
Příklad 3



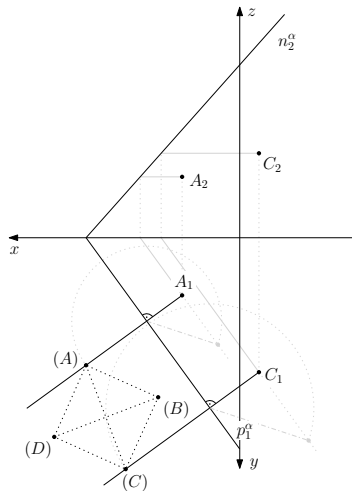
Príklad 3



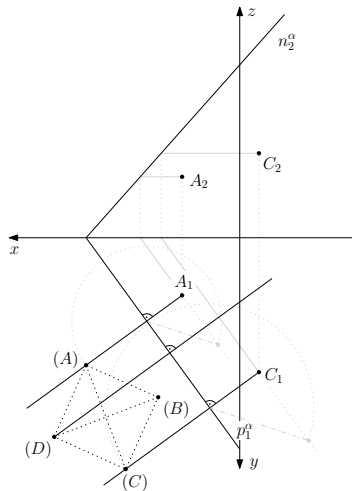
Príklad 3



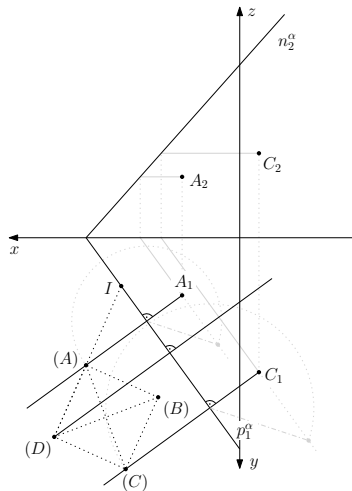
Příklad 3



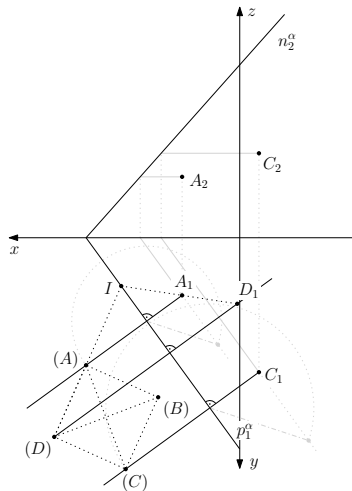
Príklad 3



Príklad 3

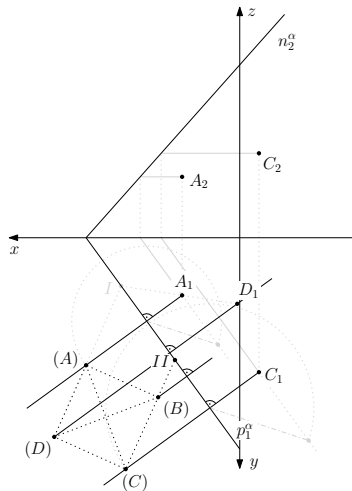


Příklad 3

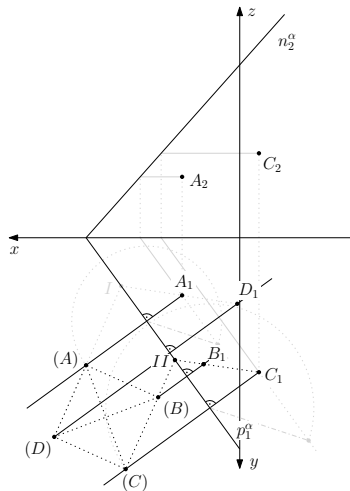


The diagram illustrates a 3D coordinate system with axes x , y , and z . A line l is shown passing through points A_1 , B , C , and D . A plane α_1 is shown, containing points C_1 , D_1 , and p_1^α . Points A_2 and C_2 are also marked in the upper part of the diagram. The diagram is used to illustrate the geometric relationships between the line l and the plane α_1 in the context of the proof.

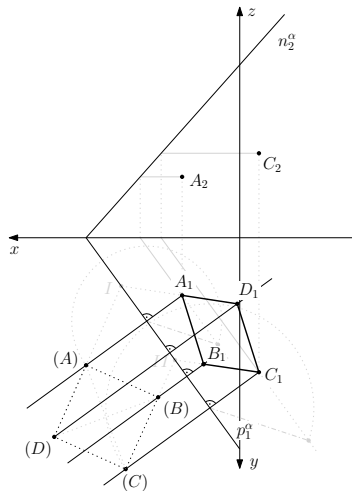
Příklad 3

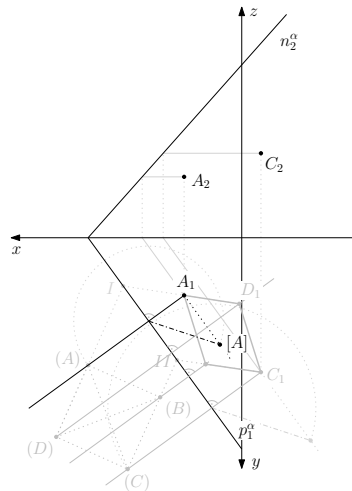


Příklad 3

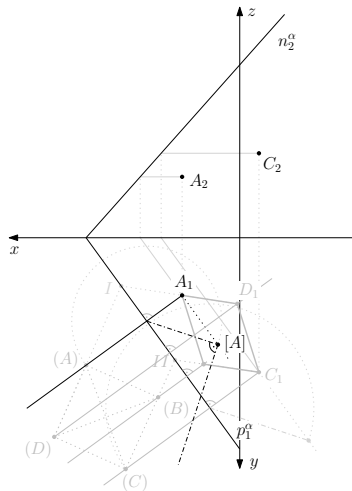


Příklad 3

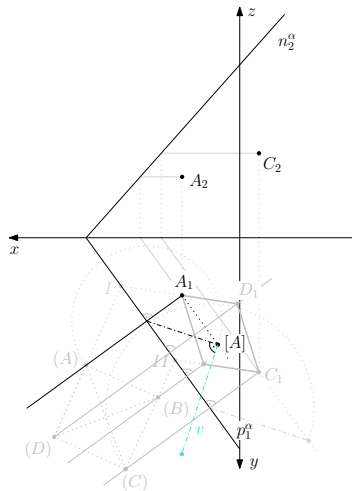




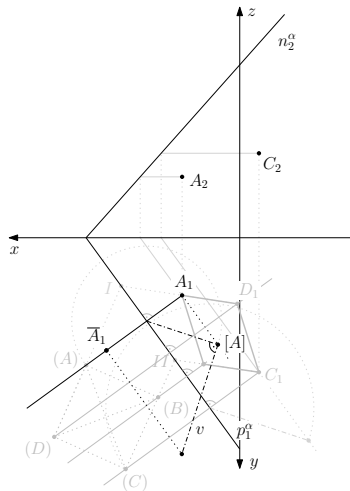
Příklad 3



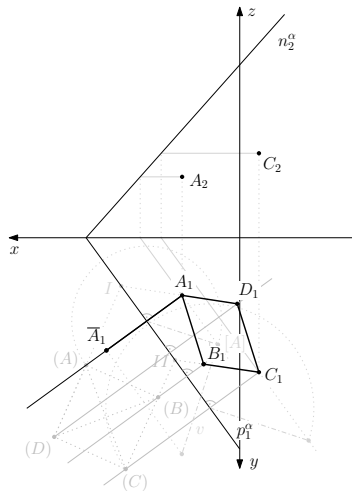
Příklad 3



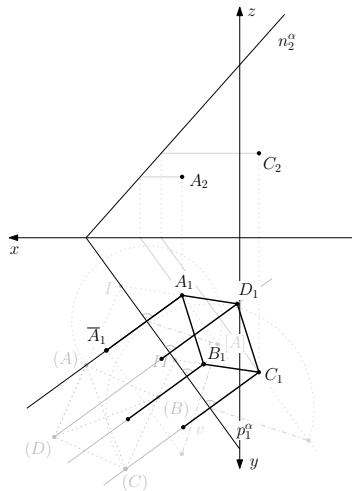
Příklad 3



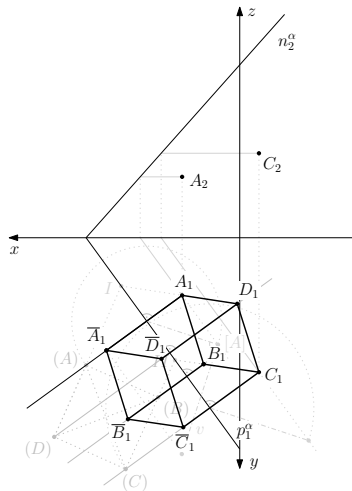
Příklad 3



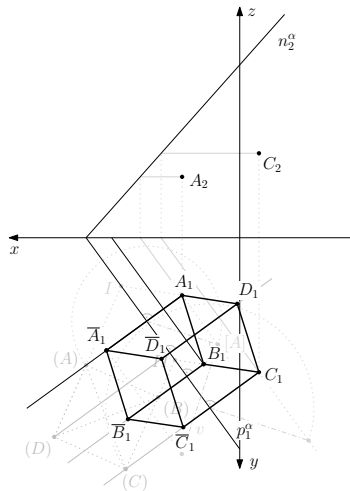
Příklad 3

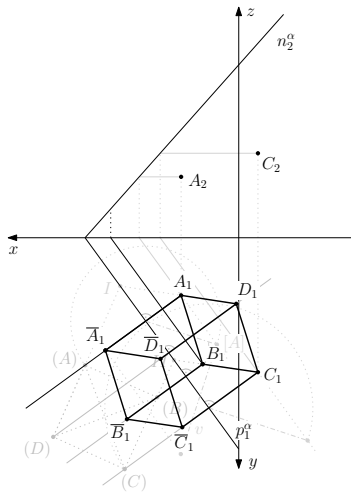


Příklad 3

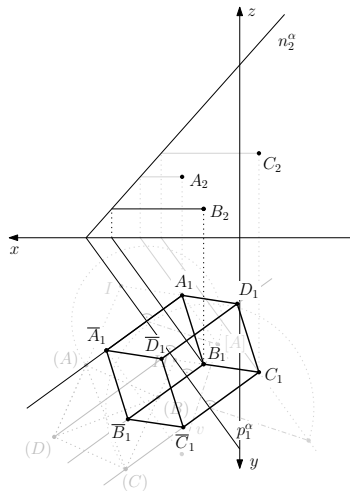


Příklad 3

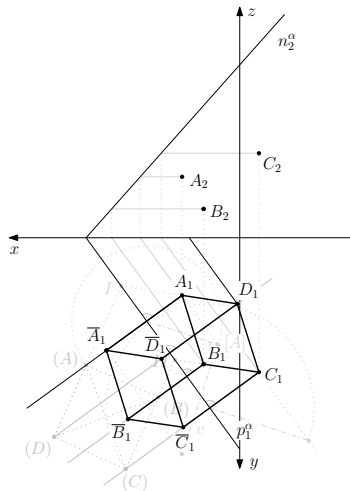




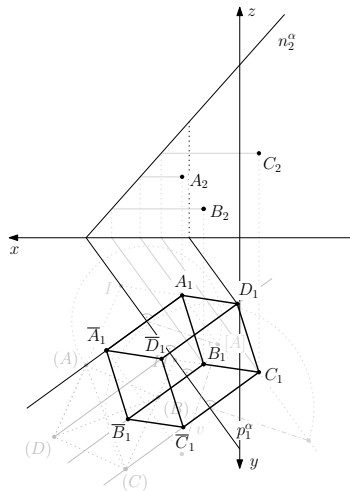
Příklad 3

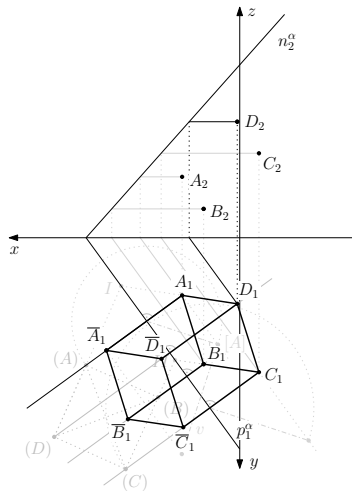


Příklad 3

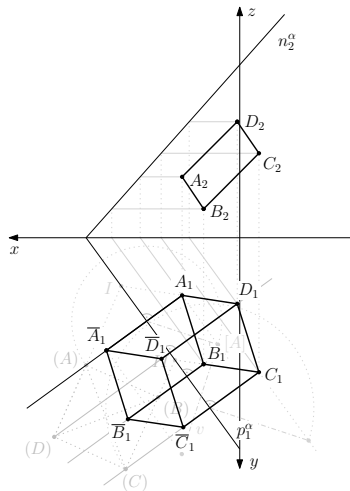


Příklad 3

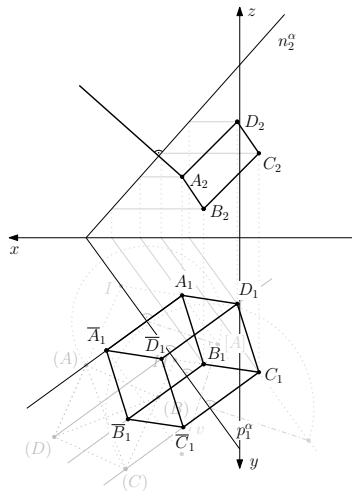




Příklad 3

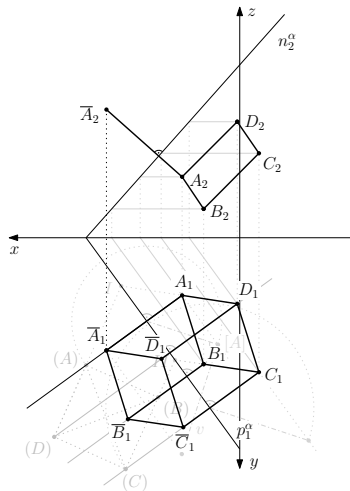


Příklad 3

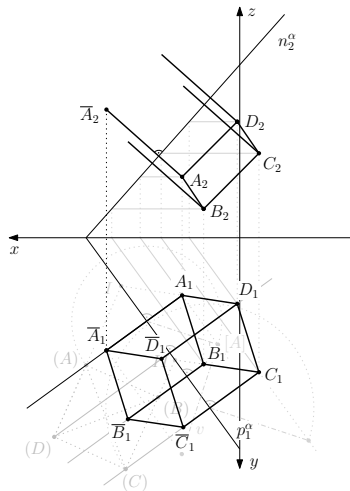


The diagram shows a 3D coordinate system with axes \$x\$, \$y\$, and \$z\$. Two tetrahedra are depicted. The first tetrahedron, \$A_1B_1C_1D_1\$, has vertices \$A_1, B_1, C_1\$ in the lower half-space (\$y < 0\$) and \$D_1\$ on the \$z\$-axis. The second tetrahedron, \$A_2B_2C_2D_2\$, has vertices \$A_2, B_2, C_2\$ in the upper half-space (\$y > 0\$) and \$D_2\$ on the \$z\$-axis. A dashed line connects \$D_1\$ and \$D_2\$. A point \$\bar{A}_2\$ is shown on a line passing through \$A_2\$ and perpendicular to the plane containing \$A_1, B_1, C_1\$. A vector \$\vec{D}_1\$ points from \$D_1\$ towards the origin. Other labels include \$(A)\$, \$(B)\$, \$(C)\$, \$(D)\$ near the bottom, and \$n_2^\alpha\$ near the top right.

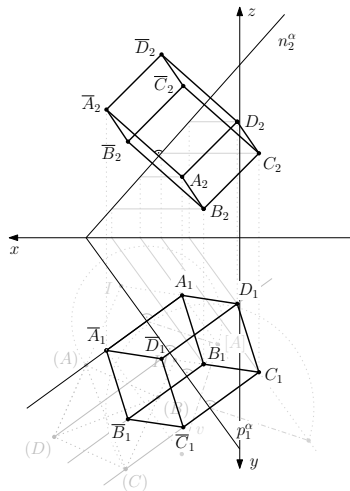
Příklad 3



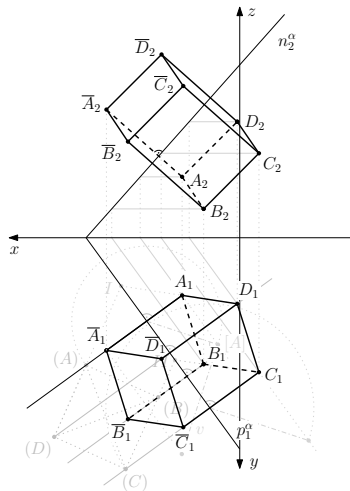
Příklad 3



Příklad 3



Příklad 3



Príklady

Vypracovať:

- ▶ **Mongeovo promítání - základní úlohy polohové**
- ▶ **Mongeovo promítání - základní úlohy metrické**