

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad x > 0, n \in \mathbf{R}, n \neq -1.$
2. $\int \frac{1}{x} dx = \ln |x| + C, \quad x \neq 0.$
3. $\int e^x dx = e^x + C.$
4. $\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0, a \neq 1.$
5. $\int \sin x dx = -\cos x + C.$
6. $\int \cos x dx = \sin x + C.$
7. $\int \frac{1}{\sin^2 x} dx = -\cotg x + C, \quad x \neq k\pi, k \in \mathbf{Z}.$
8. $\int \frac{1}{\cos^2 x} dx = \tg x + C, \quad x \neq \frac{\pi}{2} + k\pi, k \in \mathbf{Z}.$
9. $\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + C_1 = -\operatorname{arccotg} x + C_2.$
10. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C_1 = -\arccos x + C_2, \quad x \in (-1, 1).$
11. $\int \frac{1}{\sqrt{x^2+a}} dx = \ln(x + \sqrt{x^2+a}) + C, \quad a > 0.$
12. $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C, \quad a \neq 0.$
13. $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)|.$

Přímá integrace - časté využití základního vzorce $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$

1. $\int \frac{5x^2-3}{\sqrt{x}} dx.$
2. $\int \left(3x^2 - 2x + \frac{1}{\sqrt{x^3}}\right) dx.$
3. $\int \frac{3^x \cos^2 x - 5}{\cos^2 x} dx.$
4. $\int \cotg x dx.$
5. $\int \frac{\sqrt{x^4+2+x^{-4}}}{x^3} dx.$
6. $\int \frac{1}{x \ln x} dx.$
7. $\int x(x-2)(x-3) dx.$

Integrace pomocí substituce - často nutno využít vzorce $\sin^2 x = \frac{1-\cos 2x}{2}, \cos^2 x = \frac{1+\cos 2x}{2}$

8. $\int \frac{\sin^2 x}{\cos^4 x} dx;$ $[\frac{1}{3} \tg^3 x, (\text{substitute: } \tg x = t)]$
9. $\int \frac{\sqrt[3]{\operatorname{arctg} x}}{1+x^2} dx;$ $[\frac{3}{4} \sqrt[3]{(\operatorname{arctg} x)^4}, (\text{substitute: } \operatorname{arctg} x = t)]$
10. $\int \frac{1}{\sqrt{4x+9}} dx;$ $[\frac{1}{2} \sqrt{4x+9}, (\text{substitute: } 4x+9 = t)]$
11. $\int \frac{1}{7x-9} dx;$ $[\frac{1}{7} \ln |7x-9|, (\text{substitute: } 7x-9 = t)]$
12. $\int e^x \cos(e^x) dx;$ $[\sin(e^x), (\text{substitute: } e^x = t)]$
13. $\int \frac{e^{\frac{1}{x}}}{x^2} dx;$ $[-e^{\frac{1}{x}}, (\text{substitute: } \frac{1}{x} = t)]$
14. $\int \frac{x^3}{\sqrt{1-x^8}} dx;$ $[\frac{1}{4} \arcsin x^4, (\text{substitute: } x^4 = t)]$
15. $\int 2x \sqrt{x^2+1} dx;$ $[\frac{2}{3} \sqrt{(x^2+1)^3}, (\text{substitute: } x^2+1 = t^2)]$
16. $\int \frac{1}{x \cdot \ln x \cdot \ln(\ln x)} dx;$ $[\ln |\ln(\ln x)|]$

$$17. \int \frac{2^x dx}{\sqrt{1+4^x}}; \quad \left[\frac{1}{\ln 2} \ln(2^x + \sqrt{1+4^x}), (\text{substituce: } 2^x = t) \right]$$

Integrace per partes - umět odvodit vzorec $\int u'v dx = uv - \int uv' dx$ z derivace součinu uv

$$18. \int x^2 \cos x dx; \quad [(x^2 - 2) \sin x + 2x \cos x]$$

$$19. \int x^2 \operatorname{arctg} x dx; \quad \left[\frac{x^3}{3} \operatorname{arctg} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(x^2 + 1) \right]$$

$$20. \int \sin^2 x dx; \quad \left[\frac{1}{2} (x - \sin x \cos x) \right]$$

$$21. \int \frac{x}{\sin^2 x} dx; \quad [-x \cot x + \ln |\sin x|]$$

$$22. \int e^x \cos x dx; \quad \left[\frac{e^x}{2} (\sin x + \cos x) \right]$$

$$23. \int \arcsin x dx; \quad [x \arcsin x + \sqrt{1-x^2}]$$

$$24. \int x^2 \sin(2x) dx; \quad \left[-\frac{x^2}{2} \cos(2x) + \frac{x}{2} \sin(2x) + \frac{1}{4} \cos(2x) \right]$$

$$25. \int x^3 \cos x dx; \quad [(x^3 - 6x) \sin x + (3x^2 - 6) \cos x]$$

$$26. \int \cos(\ln x) dx; \quad \left[\frac{x}{2} (\cos(\ln x) + \sin(\ln x)) \right]$$

Integrace racionální lomené funkce - nejprve nutné rozložit na parciální zlomky

$$27. \int \frac{6}{x+5} dx;$$

$$28. \int \frac{-2}{(x-8)^2} dx;$$

$$29. \int \frac{3x}{x^2+4} dx;$$

$$30. \int \frac{5}{x^2+2} dx;$$

$$31. \int \frac{6x}{x^2+x+2} dx;$$

$$32. \int \frac{-2}{x^2+2x+8} dx;$$

$$33. \int \frac{x^4+6x^2+x-2}{x^4-2x^3} dx; \quad [x - 3 \ln|x| - \frac{1}{2x^2} + 5 \ln|x-2|]$$

$$34. \int \frac{5}{(2x-3)^3} dx; \quad \left[-\frac{5}{4} \frac{1}{(2x-3)^2} \right]$$

$$35. \int \frac{27 dx}{\sqrt{2x-5}}; \quad \left[\frac{27}{\sqrt{2}} \ln |\sqrt{2x-5}| \right]$$

$$36. \int \frac{8x-31}{x^2-9x+14} dx; \quad [3 \ln|x-2| + 5 \ln|x-7|]$$

$$37. \int \frac{11x^2-2x-33}{x^2-3} dx; \quad [-\ln|x-\sqrt{3}| - \ln|x+\sqrt{3}| + 11x]$$

$$38. \int \frac{4x^2+4x-11}{(2x-1)(2x+3)(2x-5)} dx; \quad \left[\frac{1}{8} \ln \left| \frac{(2x-1)^3(2x-5)^3}{2x+3} \right| \right]$$

$$39. \int \frac{4-4x}{4x^2-4x+1} dx; \quad [-\ln|2x-1| + \frac{1}{2x-1}]$$

$$40. \int \frac{6x+6}{2x^2+3x} dx; \quad [\ln|2x^3+3x^2|]$$