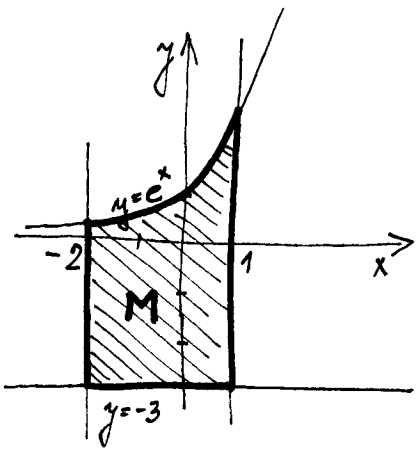
DVOJNÝ INTEGRÁL (BEZ TRANSFORMACE)

19.4.2017
UM FSI VUT

PŘ. Dvojným integrálem spočtěte obsah plochy M ohraničené křivkami
$y = e^x$, $y = -3$, $x = -2$, $x = 1$



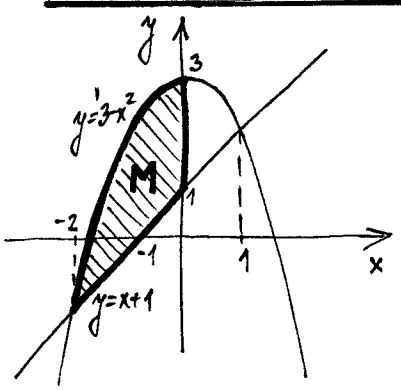
$M: \; -2 \le x \le 1$
$\quad -3 \le y \le e^x$

$$S(M) = \iint_M 1 \, dx\,dy = \int_{-2}^{1} \left(\int_{-3}^{e^x} 1 \, dy \right) dx = \int_{-2}^{1} \Big[y \Big]_{-3}^{e^x} dx =$$

$$= \int_{-2}^{1} e^x - (-3) \, dx = \Big[e^x + 3x \Big]_{-2}^{1} = e^1 + 3\cdot 1 - \left(e^{-2} + 3\cdot(-2) \right) =$$

$$= e - \frac{1}{e^2} + 9 \doteq 11{,}6$$

PŘ. Dvojným integrálem spočtěte hmotnost homogenní plochy M ohraničené křivkami $y = 3 - x^2$, $y = x + 1$, kde $x \le 0$. Plošná hustota $\sigma(x,y) = 5$.



Vypočteme průsečíky přímky a paraboly:

$$x + 1 = 3 - x^2$$
$$x^2 + x - 2 = 0$$
$$(x-1)(x+2) = 0$$
$$x = 1 \quad \vee \quad x = -2$$

$M: \; -2 \le x \le 0$
$\quad x + 1 \le y \le 3 - x^2$

$$m(M) = \iint_M \sigma(x,y) \, dx\,dy = \iint_M 5 \, dx\,dy = 5 \cdot \int_{-2}^{0} \left(\int_{x+1}^{3-x^2} 1 \, dy \right) dx = 5 \cdot \int_{-2}^{0} \Big[y \Big]_{x+1}^{3-x^2} dx =$$

$$= 5 \cdot \int_{-2}^{0} 3 - x^2 - (x+1) \, dx = 5 \cdot \int_{-2}^{0} -x^2 - x + 2 \, dx = 5 \cdot \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_{-2}^{0} =$$

$$= 5 \cdot \left(-\frac{0^3}{3} - \frac{0^2}{2} + 2\cdot 0 - \left(-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 2\cdot(-2) \right) \right) = 5 \cdot \left(-\frac{8}{3} + \frac{4}{2} + 4 \right) = \frac{50}{3} \doteq 16{,}7$$