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ÚM FSI VUT

1/2

DVOJNÝ INTEGRÁL (S TRASFORMACÍ DO POLÁRNÍCH SOUŘADNIC)

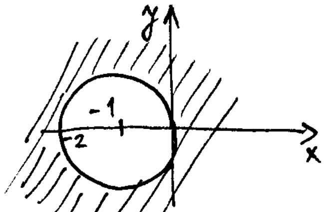
Spočtěte statický moment množiny M vzhledem ke ose x, je-li M dána nerovnostmi $x^2+y^2 \geq -2x$, $x^2+y^2 \leq -4x$, $y \leq -x$. M je homogenní, nechť $\sigma(x,y)=1$.

$x^2+y^2 \geq -2x$

$x^2+2x+y^2 \leq 0$

$(x+1)^2+y^2 \leq 1$

$S[-1,0],\ r=\sqrt{1}=1$

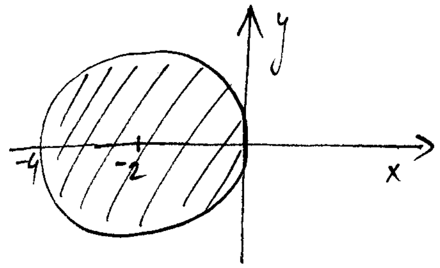


$x^2+y^2 \leq -4x$

$x^2+4x+y^2 \leq 0$

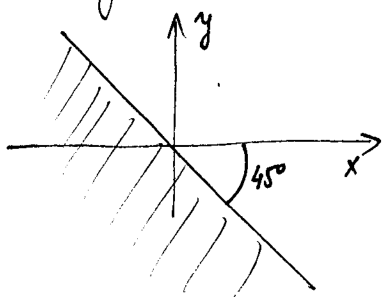
$(x+2)^2+y^2 \leq 4$

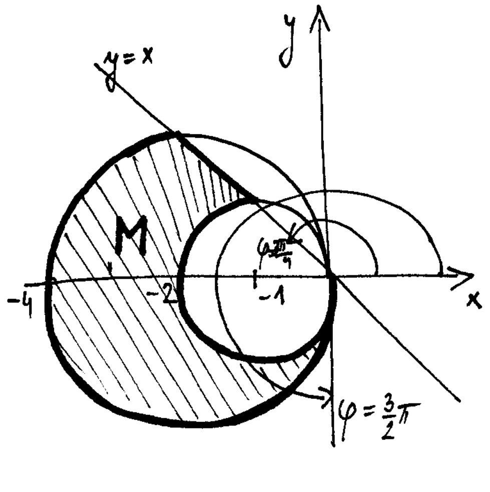
$S[-2,0],\ r=\sqrt{4}=2$



$y \leq -x$

$\operatorname{tg}\alpha = -1$





Vzhledem k tvaru množiny M bude výhodné transformovat do polárních souřadnic:

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

jakobián $= \rho$

M: $\overset{①}{-2\cos\varphi \leq} \rho \overset{②}{\leq} -4\cos\varphi$

$\frac{3\pi}{4} \leq \varphi \leq \frac{3}{2}\pi$

① $x^2+y^2 \geq -2x$

$(\rho\cos\varphi)^2+(\rho\sin\varphi)^2 \geq -2\rho\cos\varphi$

$\rho^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1} \geq -2\rho\cos\varphi$

$\rho^2 \geq -2\rho\cos\varphi$

$\rho \geq -2\cos\varphi$

② $x^2+y^2 \leq -4x$

$\vdots$

$\rho \leq -4\cos\varphi$

DVOJNÝ INTEGRÁL (S TRANSFORMACÍ DO POLÁRNÍCH SOUŘ.)

$$S_x(M) = \iint_M y \cdot \sigma(x,y)\, dx\, dy = \iint_M y \cdot 1\, dx\, dy = \int_{\frac{3}{4}\pi}^{\frac{3}{2}\pi} \left(\int_{-2\cos\varphi}^{-4\cos\varphi} \underbrace{\varrho \sin\varphi}_{y} \cdot \underset{\text{jakobián}}{\varrho}\, d\varrho \right) d\varphi =$$

$$= \int_{\frac{3}{4}\pi}^{\frac{3}{2}\pi} \sin\varphi \left(\int_{-2\cos\varphi}^{-4\cos\varphi} \varrho^2\, d\varrho \right) d\varphi = \int_{\frac{3}{4}\pi}^{\frac{3}{2}\pi} \sin\varphi \left[\frac{\varrho^3}{3} \right]_{-2\cos\varphi}^{-4\cos\varphi} d\varphi =$$

$$= \int_{\frac{3}{4}\pi}^{\frac{3}{2}\pi} \sin\varphi \left(\frac{(-4\cos\varphi)^3}{3} - \frac{(-2\cos\varphi)^3}{3} \right) d\varphi = -\frac{56}{3} \int_{\frac{3}{4}\pi}^{\frac{3}{2}\pi} \cos^3\varphi \cdot \sin\varphi\, d\varphi =$$

$$= \left| \begin{array}{l} \text{subs.} \\ \cos\varphi = t \\ -\sin\varphi\, d\varphi = dt \\ \frac{3}{4}\pi \xrightarrow{\cos\frac{3}{4}\pi} -\frac{\sqrt{2}}{2} \\ \frac{3}{2}\pi \xrightarrow{\cos\frac{3}{2}\pi} 0 \end{array} \right| = \frac{56}{3} \int_{-\frac{\sqrt{2}}{2}}^{0} t^3\, dt = \frac{56}{3} \left[\frac{t^4}{4} \right]_{-\frac{\sqrt{2}}{2}}^{0} =$$

$$= \frac{56}{3} \cdot \left(\frac{0^4}{4} - \frac{\left(-\frac{\sqrt{2}}{2}\right)^4}{4} \right) = -\frac{56}{3} \cdot \frac{\cancel{4}}{16 \cdot \cancel{4}} = \underline{\underline{-\frac{7}{6}}} \doteq -1{,}7$$