# TROJNÝ INTEGRÁL (S TRANSFORMACÍ DO SFÉRICKÝCH SOUŘADNIC)
## (OBJEM)

ÚM FSI VUT, 7.4.2017

Spočtěte objem tělesa $T$.

$T: \; 1 \leq x^2+y^2+z^2 \leq 16, \quad z \geq -\sqrt{x^2+y^2}, \quad y \leq 0, \quad y \leq -x.$

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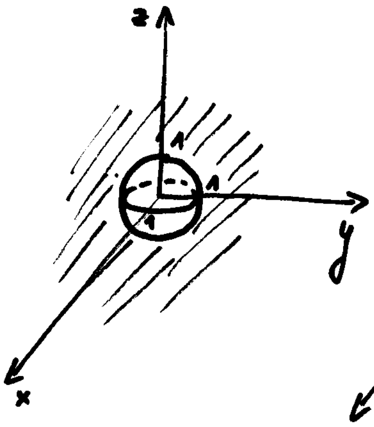


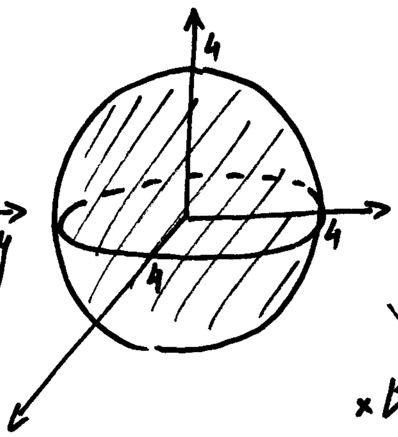


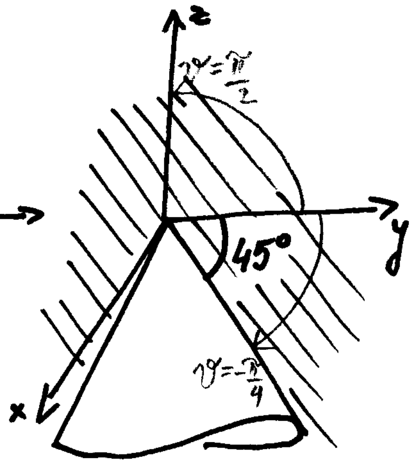


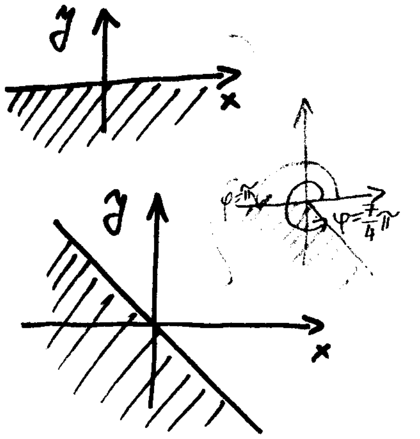


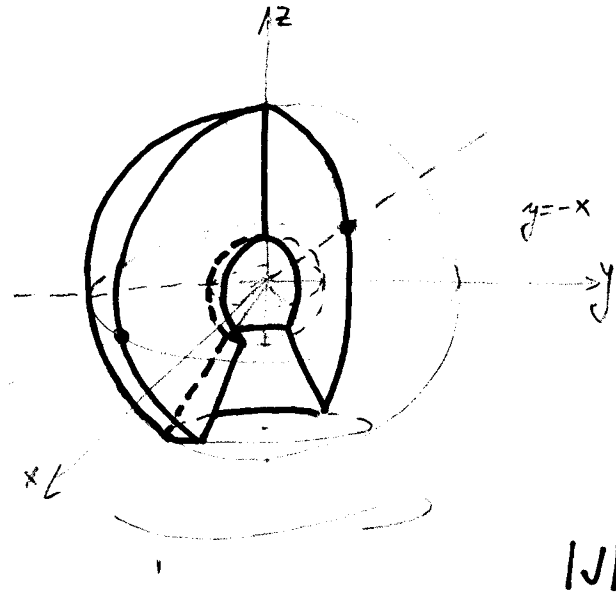


Transformace do sférických souřadnic (volím např. variantu s odklonem úhlu $\vartheta$ od půdorysny):

$$x = \varrho\cos\varphi\cdot\cos\vartheta \qquad 1 \leq \varrho \leq 4$$

$$y = \varrho\sin\varphi\cdot\cos\vartheta \qquad \pi \leq \varphi \leq \frac{7}{4}\pi$$

$$z = \varrho\sin\vartheta \qquad -\frac{\pi}{4} \leq \vartheta \leq \frac{\pi}{2}$$

$$|J| = \varrho^2\cos\vartheta$$

$$V(T) = \iiint_T 1\,dx\,dy\,dz = \int_1^4\left(\int_\pi^{\frac{7}{4}\pi}\left(\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 1\cdot\underbrace{\varrho^2\cos\vartheta}_{|\text{jakobián}|}\,d\vartheta\right)d\varphi\right)d\varrho =$$

$$= \int_1^4 \varrho^2\left(\int_\pi^{\frac{7}{4}\pi}\Big[\sin\vartheta\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{2}}d\varphi\right)d\varrho = \left(\sin\frac{\pi}{2}-\sin\left(-\frac{\pi}{4}\right)\right)\int_1^4\varrho^2\left(\int_\pi^{\frac{7}{4}\pi}1\,d\varphi\right)d\varrho =$$

$$= \left(1-\left(-\frac{\sqrt{2}}{2}\right)\right)\int_1^4\varrho^2\Big[\varphi\Big]_\pi^{\frac{7}{4}\pi}d\varrho = \left(1+\frac{\sqrt{2}}{2}\right)\left(\frac{7}{4}\pi-\pi\right)\int_1^4\varrho^2\,d\varrho =$$

$$= \left(1+\frac{\sqrt{2}}{2}\right)\cdot\frac{3}{4}\pi\left[\frac{\varrho^3}{3}\right]_1^4 = \frac{3}{4}\pi\left(1+\frac{\sqrt{2}}{2}\right)\left(\frac{4^3}{3}-\frac{1^3}{3}\right) = \underline{\underline{\frac{63}{8}(2+\sqrt{2})\pi}} \doteq 84{,}5$$