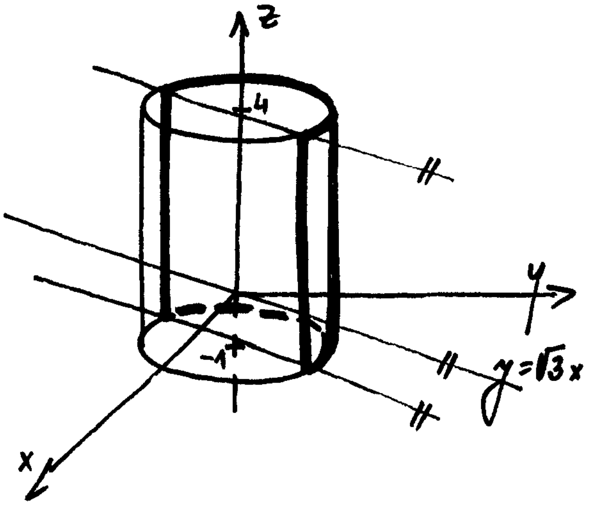
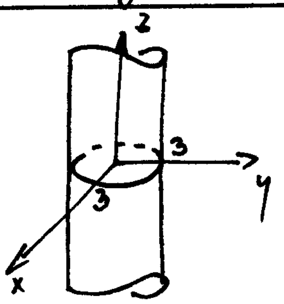
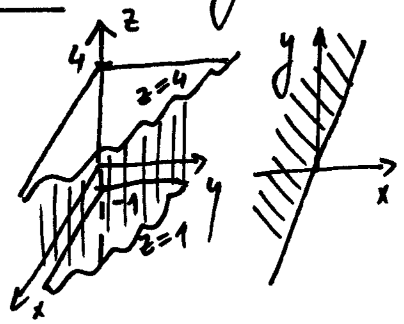
PLOŠNÝ INT. 1. DRUHU – PLOCHA DANÁ PARAMETRICKY

Spočtěte $\iint\limits_{S} y^2\, dS$, kde $S = \{[x,y,z] \in \mathbb{R}^3 : x^2 + y^2 = 9,\ -1 \le z \le 4,\ y \ge \sqrt{3}x\}$





PLOCHA



Plochu není možné vyjádřit explicitně, proto ji parametrizujeme:

$x = 3\cos u$

$y = 3\sin u$

$z = v, \qquad u \in \langle \frac{\pi}{3}, \frac{4}{3}\pi \rangle$

$\qquad\qquad v \in \langle -1, 4 \rangle$

Nachystáme si normálu $\vec{n} = \vec{t_u} \times \vec{t_v}$:

$$\vec{t_u} = \left(\frac{\partial\, 3\cos u}{\partial u}, \frac{\partial\, 3\sin u}{\partial u}, \frac{\partial v}{\partial u} \right) = (-3\sin u, 3\cos u, 0)$$

$$\vec{t_v} = \left(\frac{\partial\, 3\cos u}{\partial v}, \frac{\partial\, 3\sin u}{\partial v}, \frac{\partial v}{\partial v} \right) = (0, 0, 1)$$

$$\vec{n} = (3\cos u \cdot 1 - 0\cdot 0,\ 0\cdot 0 - (-3\sin u)\cdot 1,\ (-3\sin u)\cdot 0 - 3\cos u \cdot 0) =$$

$$= (3\cos u, 3\sin u, 0)$$

Velikost normály:

$$|\vec{n}| = \sqrt{(3\cos u)^2 + (3\sin u)^2 + 0^2} = \sqrt{9\cos^2 u + 9\sin^2 u} =$$

$$= \sqrt{9(\underbrace{\cos^2 u + \sin^2 u}_{1})} = \sqrt{9} = 3$$

PLOŠNÝ INT. 1. DRUHU – PLOCHA DANÁ PARAMETRICKY

$$\iint_{\varphi} y^2 \, dS \overset{\text{DEFINICE}}{=\!=\!=} \int_{\frac{\pi}{3}}^{\frac{4}{3}\pi} \left(\int_{-1}^{4} \underbrace{(3\sin u)}_{y}^2 \cdot \underbrace{3}_{|\vec{n}|} \, dv \right) du =$$

$$= 27 \int_{\frac{\pi}{3}}^{\frac{4}{3}\pi} \sin^2 u \left(\int_{-1}^{4} 1 \, dv \right) du \overset{\sin^2 u = \frac{1-\cos 2u}{2}}{=\!=\!=}$$

$$= 27 \int_{\frac{\pi}{3}}^{\frac{4}{3}\pi} \frac{1-\cos 2u}{2} \Big[v \Big]_{-1}^{4} du \overset{\int \cos 2u \, du = \frac{\sin 2u}{2}}{=\!=\!=}$$

$$= \frac{27}{2} \left(4-(-1)\right) \cdot \left[u - \frac{\sin 2u}{2} \right]_{\frac{\pi}{3}}^{\frac{4}{3}\pi} =$$

$$= \frac{135}{2} \left(\frac{4}{3}\pi - \frac{\sin\left(2 \cdot \frac{4}{3}\pi\right)}{2} - \left(\frac{\pi}{3} - \frac{\sin\left(2 \cdot \frac{\pi}{3}\right)}{2} \right) \right) =$$

$$= \frac{135}{2} \left(\pi - \frac{\frac{\sqrt{3}}{2}}{2} + \frac{\frac{\sqrt{3}}{2}}{2} \right) = \underline{\underline{\frac{135}{2}\pi}} \doteq 212{,}1$$