

# TROJNÝ INTEGRÁL (BEZ TRANSFORMACE)

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Pročítejte  $\iiint_T xyz \, dx \, dy \, dz$ ,  $T: 0 \leq x \leq 3, 0 \leq y \leq x+1, x-5 \leq z \leq 0$

$$\iiint_T xyz \, dx \, dy \, dz = \int_0^3 \left( \int_0^{x+1} \left( \int_{x-5}^0 xyz \, dz \right) dy \right) dx =$$

$$= \int_0^3 x \left( \int_0^{x+1} y \left[ \frac{z^2}{2} \right]_{x-5}^0 dy \right) dx = \int_0^3 x \left( \frac{(x-5)^2}{2} - \frac{0^2}{2} \right) \cdot \left[ \frac{y^2}{2} \right]_0^{x+1} dx =$$

$$= - \int_0^3 x \cdot \frac{x^2 - 10x + 25}{2} \cdot \left( \frac{(x+1)^2}{2} - \frac{0^2}{2} \right) dx = - \frac{1}{4} \int_0^3 (x^3 - 10x^2 + 25x) (x^2 + 2x + 1) dx =$$

$$= - \frac{1}{4} \int_0^3 (x^5 + 2x^4 + x^3 - 10x^4 - 20x^3 - 10x^2 + 25x^3 + 50x^2 + 25x) dx =$$

$$= - \frac{1}{4} \int_0^3 (x^5 - 8x^4 + 6x^3 + 40x^2 + 25x) dx =$$

$$= - \frac{1}{4} \left[ \frac{x^6}{6} - 8 \frac{x^5}{5} + 6 \frac{x^4}{4} + 40 \frac{x^3}{3} + 25 \frac{x^2}{2} \right]_0^3 =$$

$$= - \frac{1}{4} \left( \frac{3^6}{6} - \frac{8 \cdot 3^5}{5} + \frac{3 \cdot 3^4}{2} + 40 \cdot \frac{3^3}{3} + 25 \frac{3^2}{2} - 0 \right) = - \frac{3267}{40} \approx -81,7$$

