Proposition

1. The Earth moves around the Sun.
2. It rained yesterday.
3. Will you come tomorrow?
4. Dear me!
5. All the numbers are odd.
6. Give me that knife!
7. There exists a number which is less than 7.
8. Not all the people on Mars are white.

Sentences 1, 2, 5, 7, 8 are propositions while 3, 4, and 6 are not:

1 is generally accepted to be true

2 may or may not be true

5 is false

7 can easily be proved to be true by taking 6 for instance

8 may be proved to be true or false in the near future

A proposition is a statement that people can examine in order to decide whether it is true.

We shall denote propositions by capital letters:

A, *B*, *C*, ...

Tomorrow I will receive a letter or I will go for a ride

$$A \vee B$$

The ice on the lake is broken and the sun is shining.

$$A \wedge B$$

If it rains tomorrow, you will see how much dirt there is here.

$$A \Rightarrow B$$

That door creaks exactly when the hinges are not oiled and it opens

$$A \Leftrightarrow (B \wedge C)$$

It is not true that Mike won the race

$$\neg A$$

Logical "or"

A	B	$A \vee B$
1	1	1
1	0	1
0	1	1
0	0	0

Logical "and"

A	B	$A \wedge B$
1	1	1
1	0	0
0	1	0
0	0	0

Logical "if" (implication)

A	B	$A \Rightarrow B$
1	1	1
1	0	0
0	1	1
0	0	1

Equivalence

A	B	$A \Leftrightarrow B$
1	1	1
1	0	0
0	1	0
0	0	1

Negation

A	$\neg A$
1	0
0	1

Complex propositions containing several atom propositions that are true for all combinations of truth values of the atoms are called tautologies. They play a very important role in the calculus of propositions.

Example

$$(A \Rightarrow B) \Leftrightarrow (\neg A \vee B)$$

The man by the name of X is honest.

$$x > 7$$

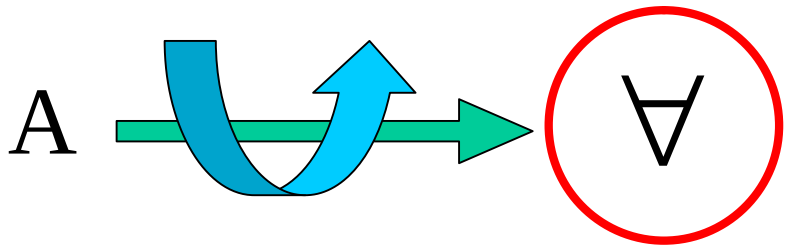
$$x^2 - 4x + 5 > 0$$

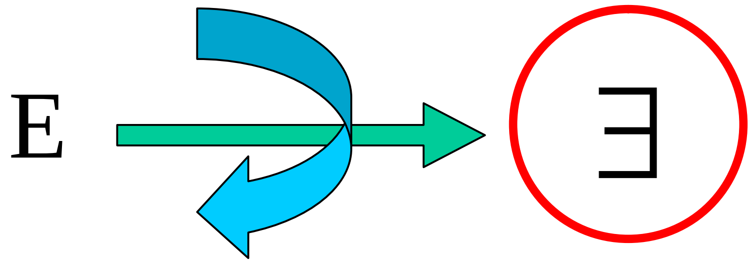
$$x^2 - 4x - 5 > 0$$

$$x + y \geq \frac{7x - y}{10}$$

Some propositions may depend on parameters or variables and their truth value may only be established after substitutions.

Another way of making propositions with variables definite is using the $\forall$ and $\exists$ quantifiers





$\exists x\ldots$ for all x we have ...

$\forall x\ldots$ there exists x such that ...

Examples:

$\forall : \left(x^2 - 4x + 5 > 0\right)$ true

$\exists x : \left(x^2 - 4x - 5 < 0\right)$ false

Negation of propositions with quantifiers

$$\neg(\forall x : P(x)) = \exists x : (\neg P(x))$$

$$\neg(\exists x : P(x)) = \forall x : (\neg P(x))$$

Example

The proposition:

"it is not true that the polynomial $p(x)$ is greater than or equal to zero for all the values of x"

amounts to the same as :

"there exists an x_0 such that $p(x_0) < 0$".