Notions such as ***set***, ***element***, ***being an element of a set*** are used as primitives or atoms in the theory of sets.

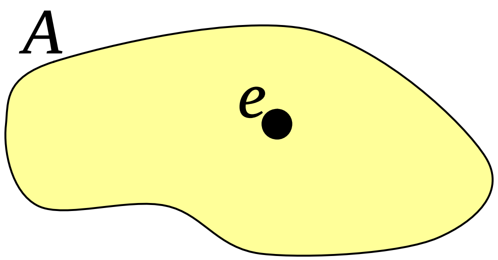
One way of defining such "atoms" is to list all the relationships betwen such atoms that we want to be true. Of course such underlying relatioships or "axioms" have to be logically consistent.

This is an axiomatic approach to the theory of sets

Instead, we choose a "naive" approach, presuming that "*everyone can tell a set*" stressing only that

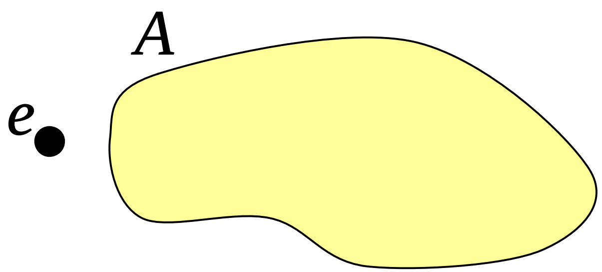
For each element *e* and set *A*, it must be clear whether *e* is a member of *A*

formally $e \in A$ graphically



or not

formally $e \notin A$ graphically



When defining a new set *S*,we often use the following notation. If $P(x)$ denotes formally the fact that *x* has property *P*, we write

$$S = \{x \mid P(x)\}$$

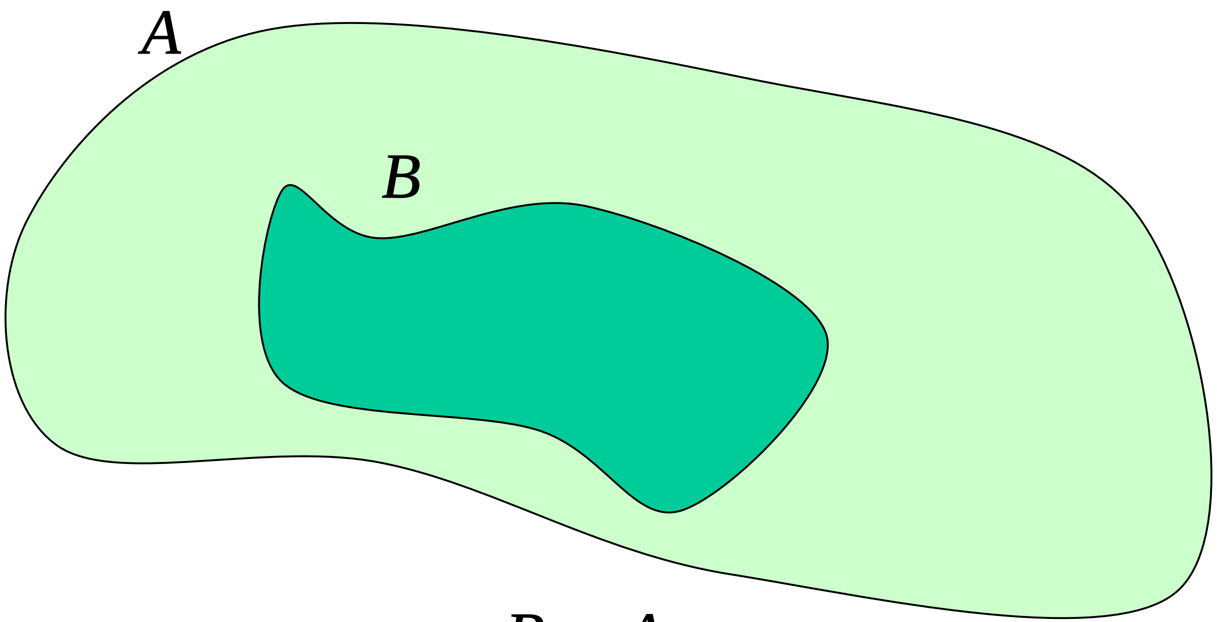
to say that *S* is the set of all those elements *x* that have property *P*.

For example

$$I = \{x \mid x \geq 1 \wedge x \leq 2\}$$

denotes the set of all points of the closed interval [1,2].

Subset



$$B \subseteq A$$

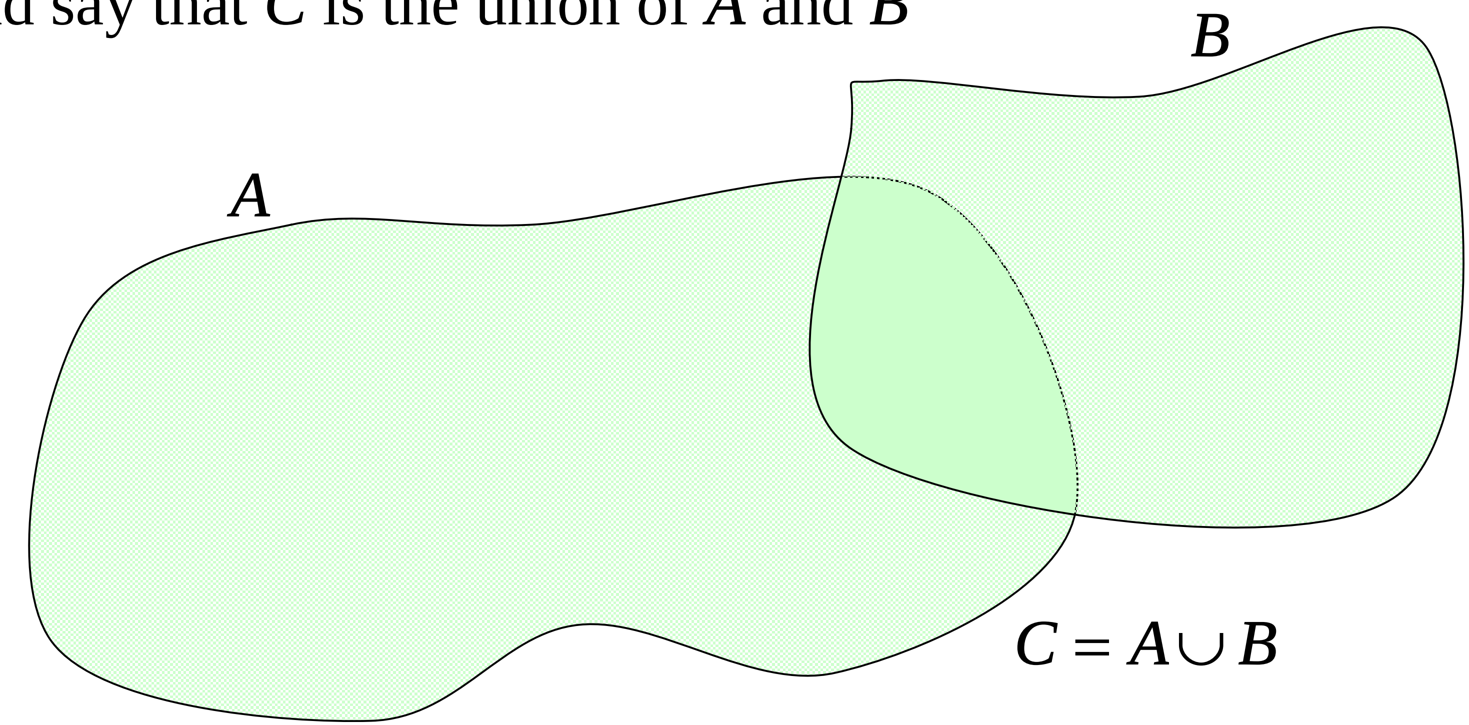
$$\forall x : (x \in B \Rightarrow x \in A)$$

We say that B is a subset of A (A is a superset of B)

Union of sets

If $C = \{x \mid x \in A \vee x \in B\}$ then we write $C = A \cup B$

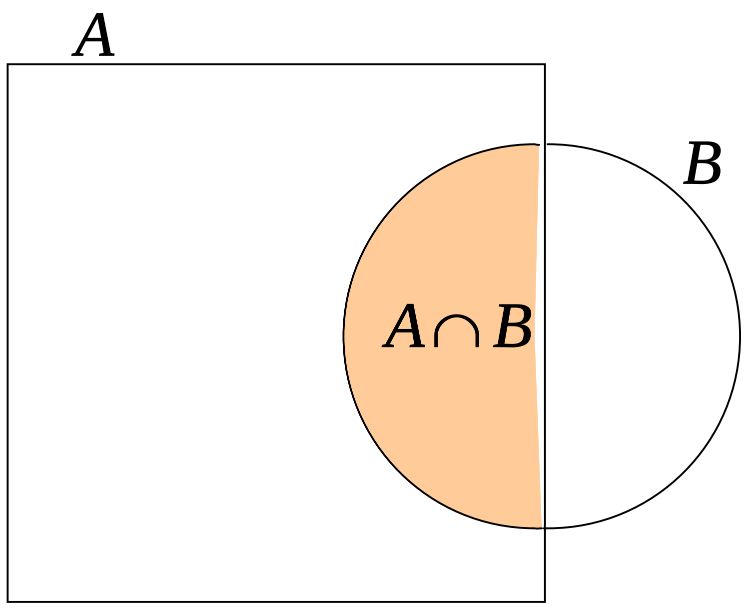
and say that C is the union of A and B



Intersection of sets

If $C = \{x \mid x \in A \wedge x \in B\}$ then we write $C = A \cap B$

and say that C is the intersection of A and B



Equal sets, proper subset

We say that the sets A and B are equal or $A = B$ if

$$A \subseteq B \wedge B \subseteq A$$

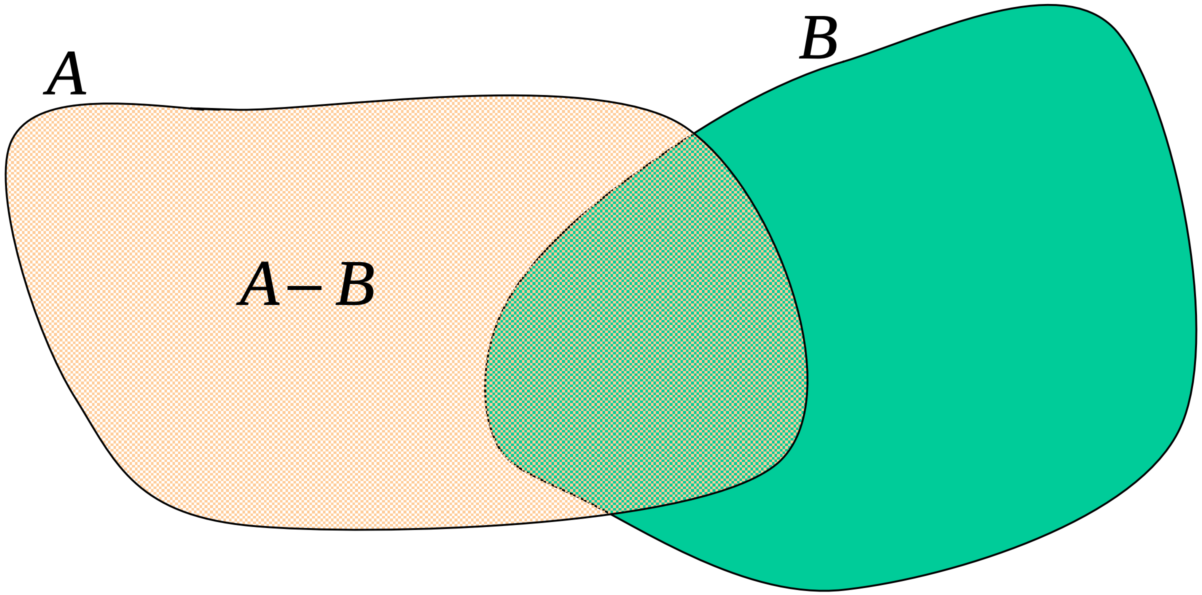
If $A \subseteq B$ and $A \neq B$ we say that A is a ***proper subset*** of B

writing $A \subset B$

Difference

The difference of the sets A and B:

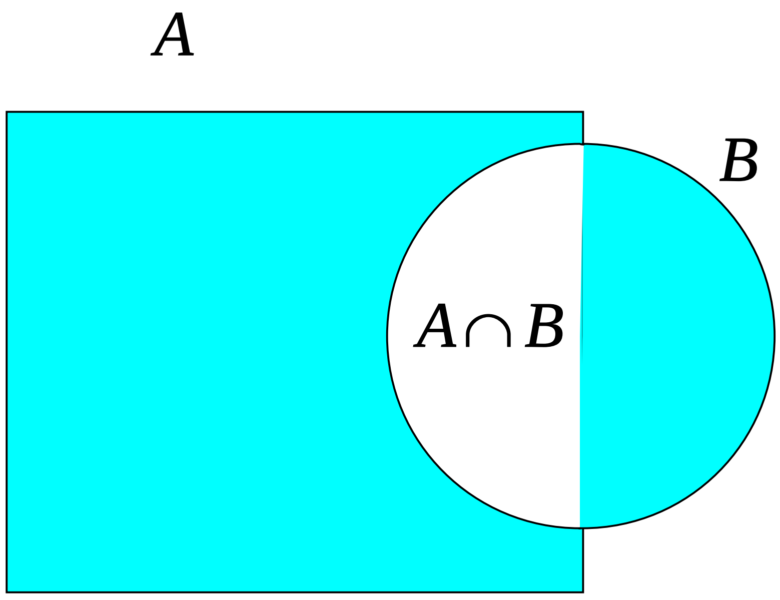
$$A - B = \{x \mid x \in A \wedge x \notin B\}$$



Symmetrical difference

The symmetrical difference of the sets A and B :

$$A \div B = (A \cup B) - (A \cap B)$$

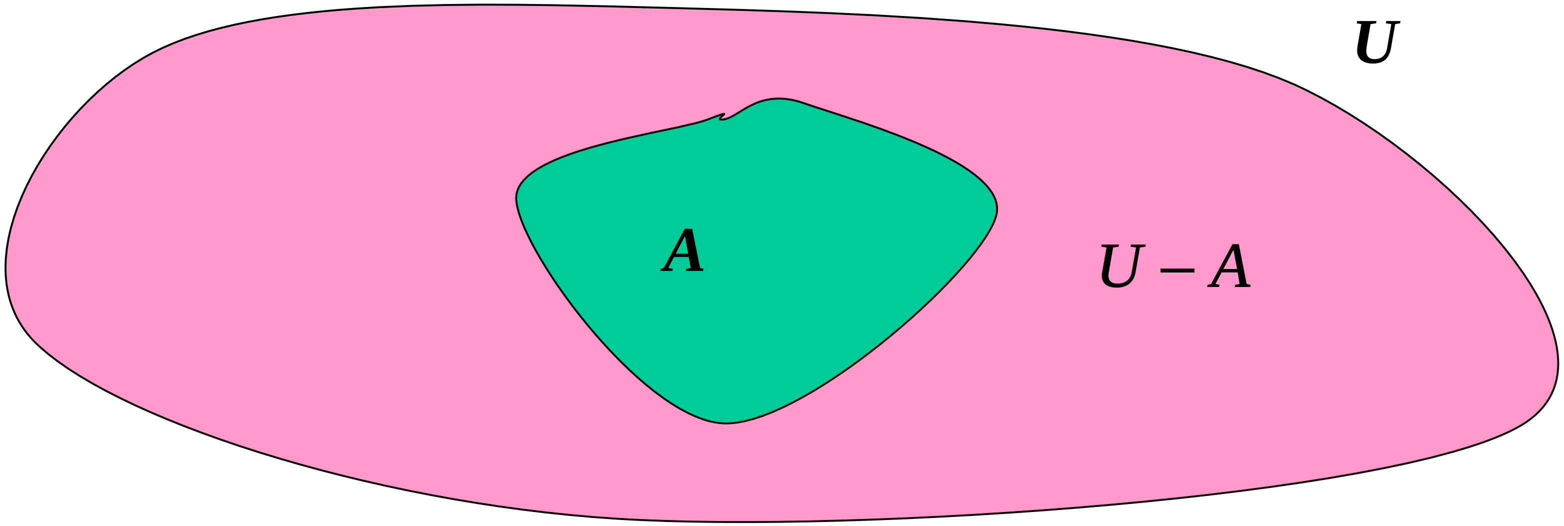


Complement

Let U be a superset of all the sets considered in a particular problem.

$$\overline{A} = U - A$$

We call $\overline{A}$ the complement of A.



De Morgan's laws

(1) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(2) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Let us prove (1):

$\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

$$x \in \overline{A \cup B} \Rightarrow x \notin A \cup B \Rightarrow x \notin A \wedge x \notin B \Rightarrow x \in \overline{A} \wedge x \in \overline{B} \Rightarrow x \in \overline{A} \cap \overline{B}$$

$\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

$$x \in \overline{A} \cap \overline{B} \Rightarrow x \in \overline{A} \wedge x \in \overline{B} \Rightarrow \neg(x \in A \vee x \in B) \Rightarrow x \notin A \cup B \Rightarrow x \in \overline{A \cup B}$$

By analogy we define unions and intersections of a finite number of sets:

$$B = A_1 \cup A_2 \cup \ldots \cup A_n \quad \text{means that} \quad B = \{x \mid \exists i : (x \in A_i)\}$$

$$B = A_1 \cap A_2 \cap \ldots \cap A_n \quad \text{means that} \quad B = \{x \mid \forall i : (x \in A_i)\}$$

Sometimes we use the notation

$$B = A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

$$B = A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$$

General form of De Morgan's laws

$$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$$

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$