Parametric and general equation of a geometrical object O

t

INPUT

EQUATION

$\left(x(t), y(t), z(t)\right) \in O$

OUTPUT

(x, y, z)

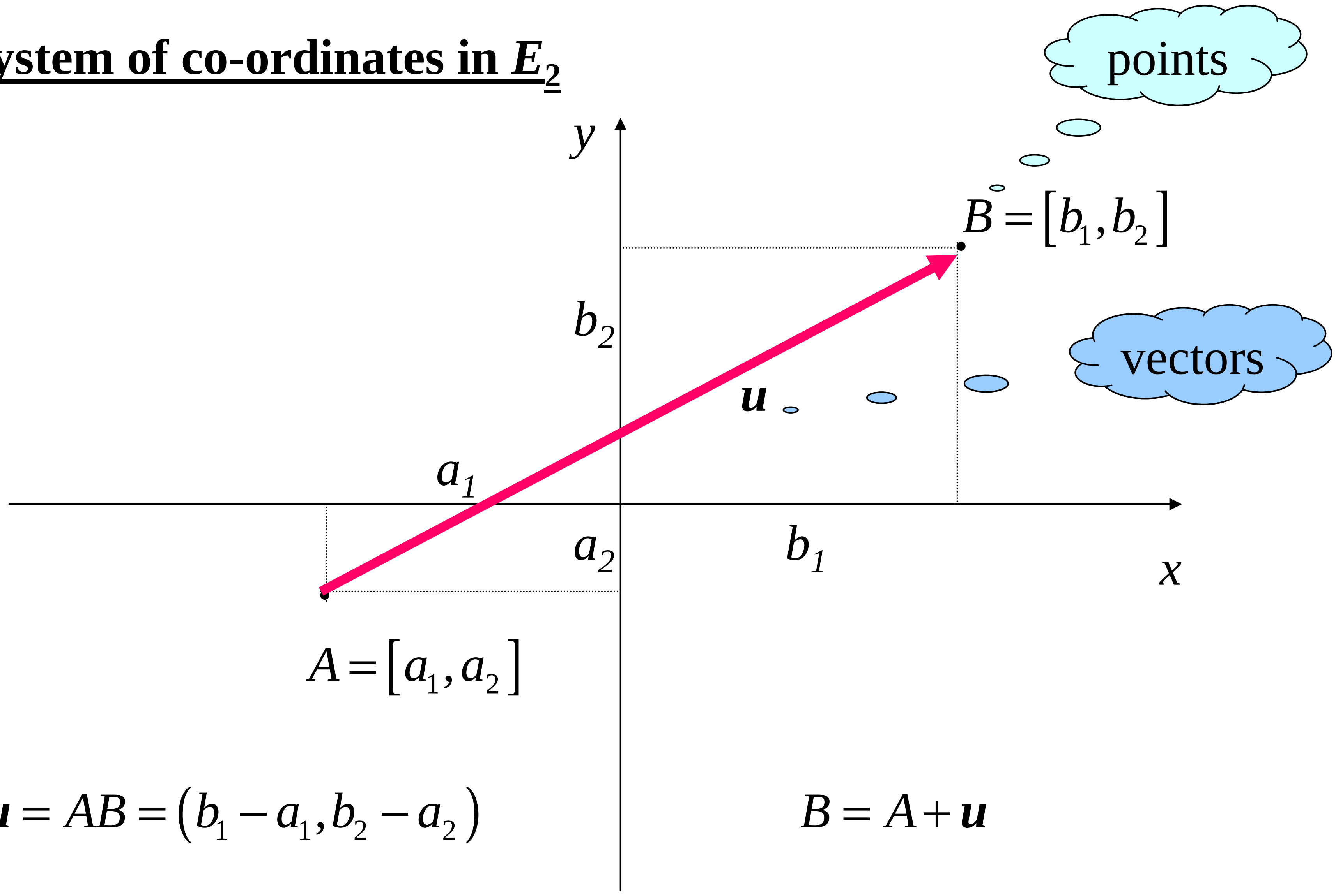
INPUT

EQUATION

OUTPUT

$$\begin{cases} (x, y, z) \in O \\ (x, y, z) \notin O \end{cases}$$

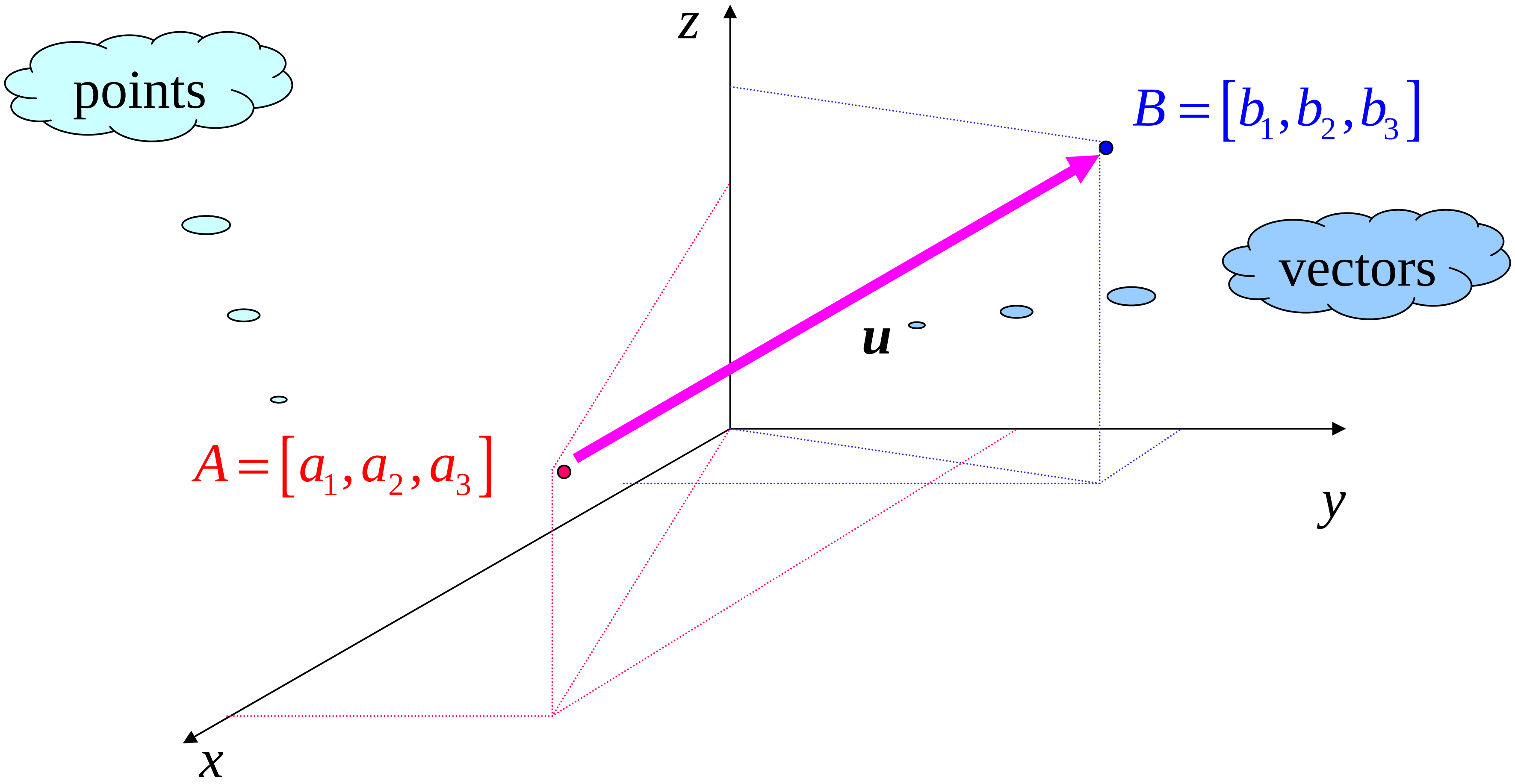
System of co-ordinates in E_2



$$\boldsymbol{u} = AB = (b_1 - a_1, b_2 - a_2)$$

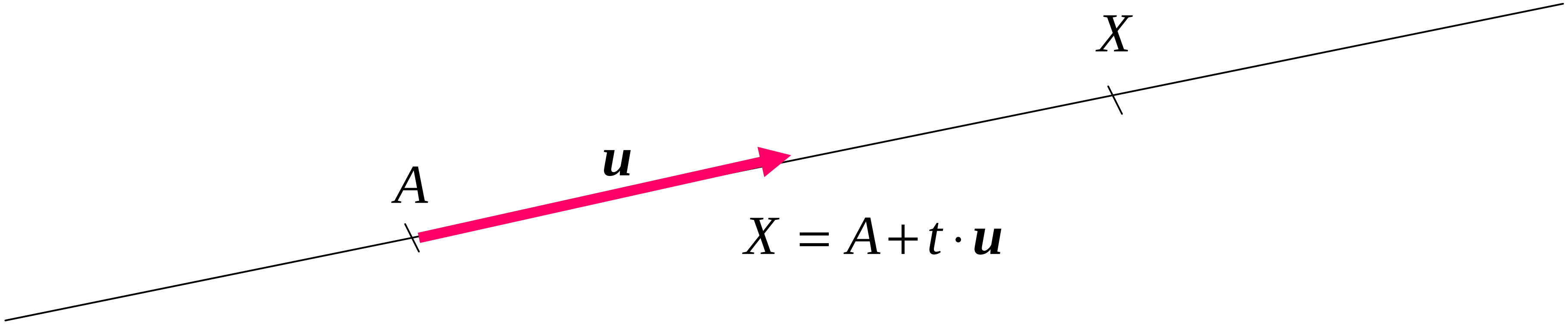
$$B = A + \boldsymbol{u}$$

System of co-ordinates in E_3



$$\boldsymbol{u} = AB = (b_1 - a_1, b_2 - a_2, b_3 - a_3) \qquad B = A + \boldsymbol{u}$$

Parametric equations of a straight line



$$X = A + t \cdot \boldsymbol{u}$$

in E_2

$$A = [a_1, a_2]$$

$$\boldsymbol{u} = (u_1, u_2)$$

$$\begin{aligned} x &= a_1 + tu_1 \\ y &= a_2 + tu_2 \end{aligned}$$

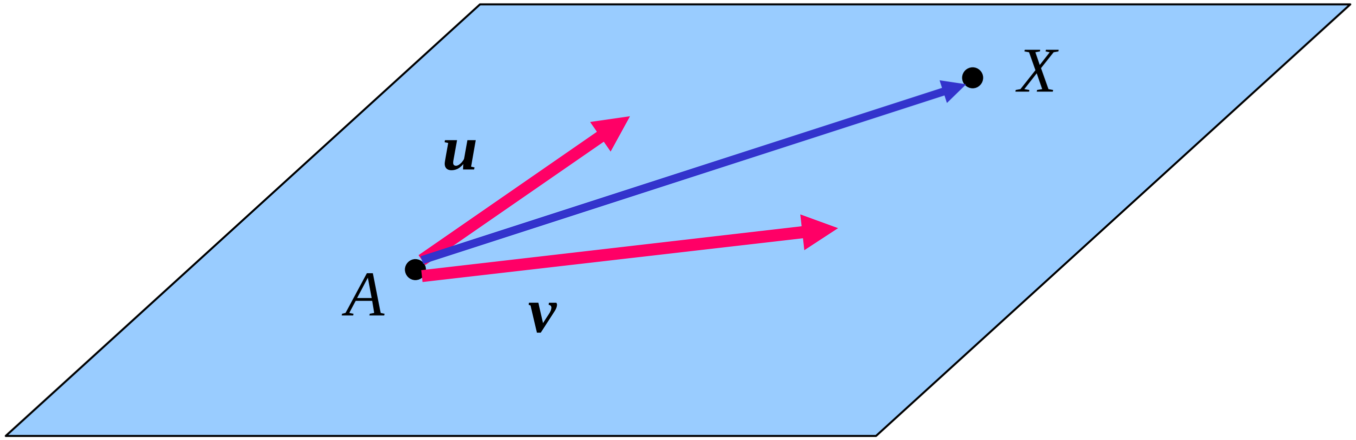
in E_3

$$A = [a_1, a_2, a_3]$$

$$\boldsymbol{u} = (u_1, u_2, u_3)$$

$$\begin{aligned} x &= a_1 + tu_1 \\ y &= a_2 + tu_2 \\ z &= a_3 + tu_3 \end{aligned}$$

Parametric equations of a plane



$$A = [a_1, a_2, a_3] \qquad X = A + s\boldsymbol{u} + t\boldsymbol{v}$$

$$\boldsymbol{u} = (u_1, u_2, u_3)$$

$$\boldsymbol{v} = (v_1, v_2, v_3)$$

$$\begin{aligned} x &= a_1 + su_1 + tv_1 \\ y &= a_2 + su_2 + tv_2 \\ z &= a_3 + su_3 + tv_3 \end{aligned}$$

General equation of a straight line in E_2

$$ax + by + c = 0$$

If a straight line is given by two points $A = [a_1, a_2], B = [b_1, b_2]$

then its general equation can be expressed as

$$\begin{vmatrix} x - a_1 & y - a_2 \\ b_1 - a_1 & b_2 - a_2 \end{vmatrix} = 0$$

General equation of a plane in E_3

$$\boxed{ax + by + cz + d = 0}$$

If a plane is given by three points

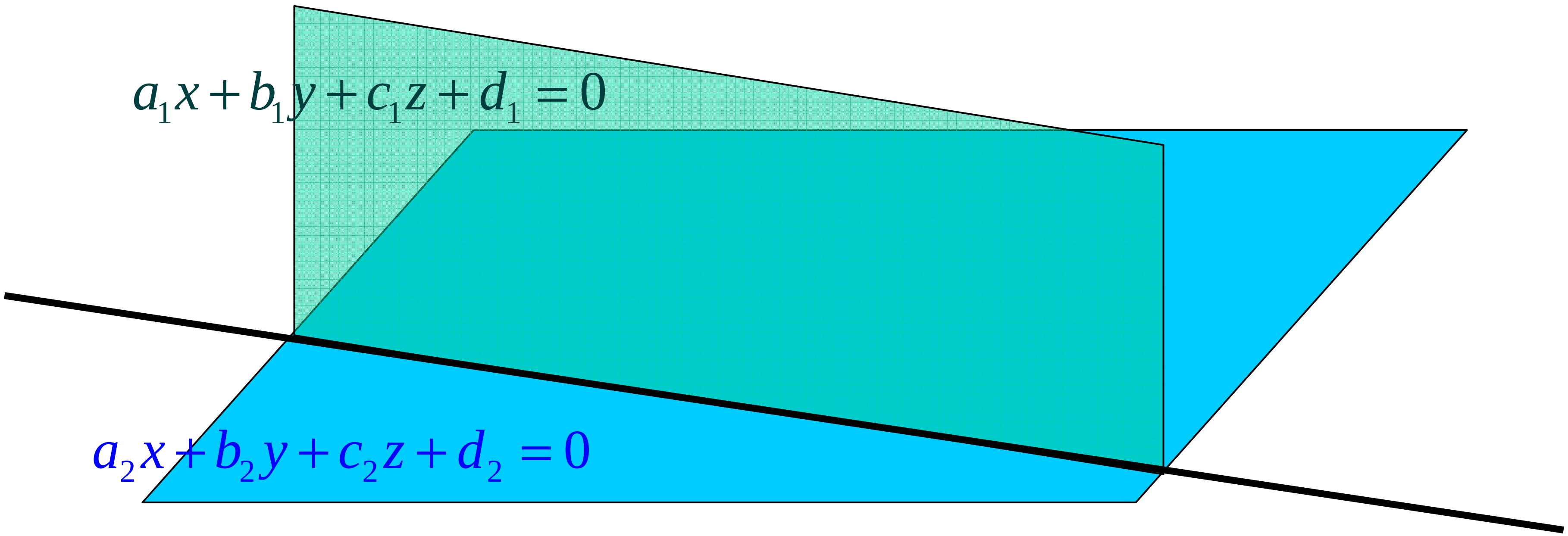
$$A = [a_1, a_2, a_3], B = [b_1, b_2, b_3], C = [c_1, c_2, c_3]$$

then its general equation can be expressed as

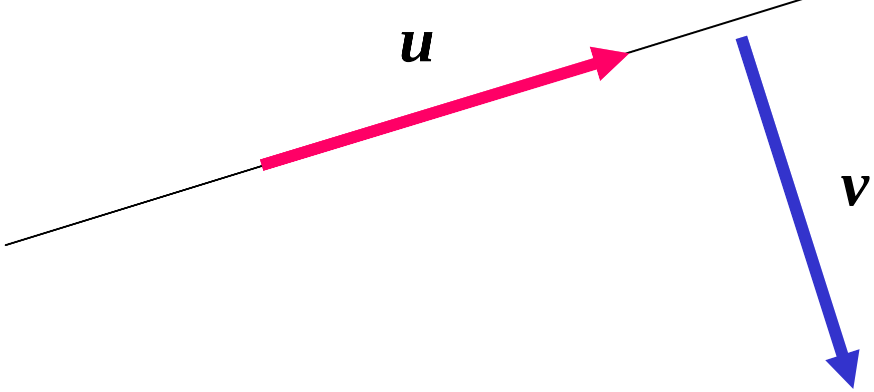
$$\begin{vmatrix} x - a_1 & y - a_2 & z - a_3 \\ b_1 - a_1 & b_2 - a_2 & b_3 - a_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{vmatrix} = 0$$

General equations of a straight line in E_3

A straight line n E_3 is defined by the general equations of two planes that intersect in that straight line.



Given a general equation of a straight line $\boxed{ax+by+c=0}$



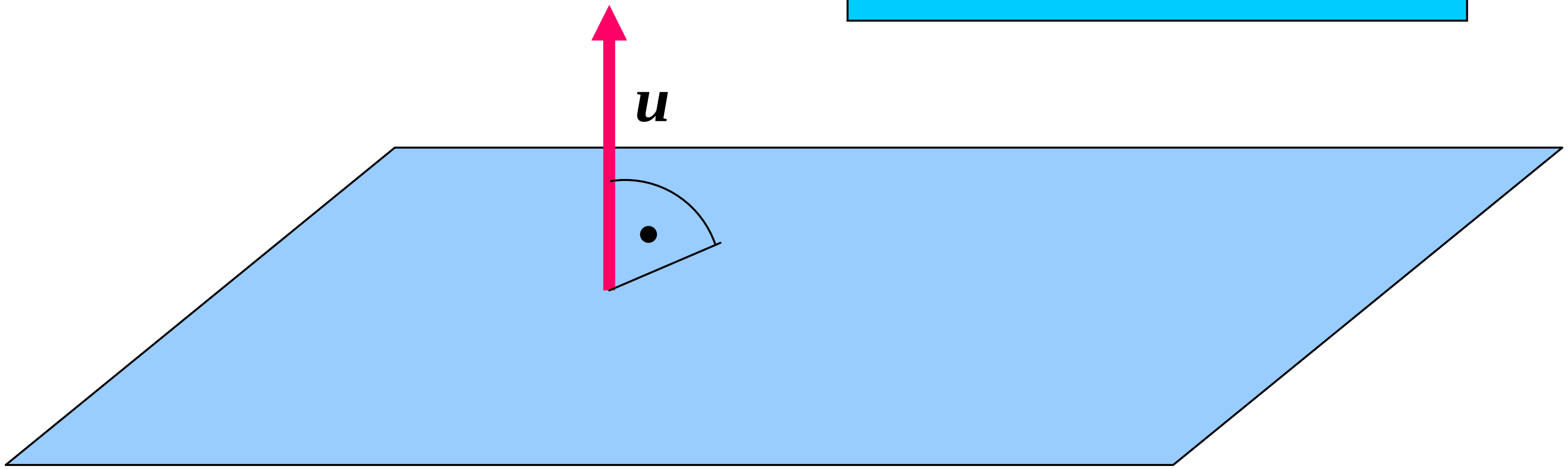
how can we determine a vector $\boldsymbol{u}$ parallel to and a vector $\boldsymbol{v}$ perpendicular to that straight line ?

$$\boldsymbol{u} = k(-b, a)$$

$$\boldsymbol{v} = k(a, b)$$

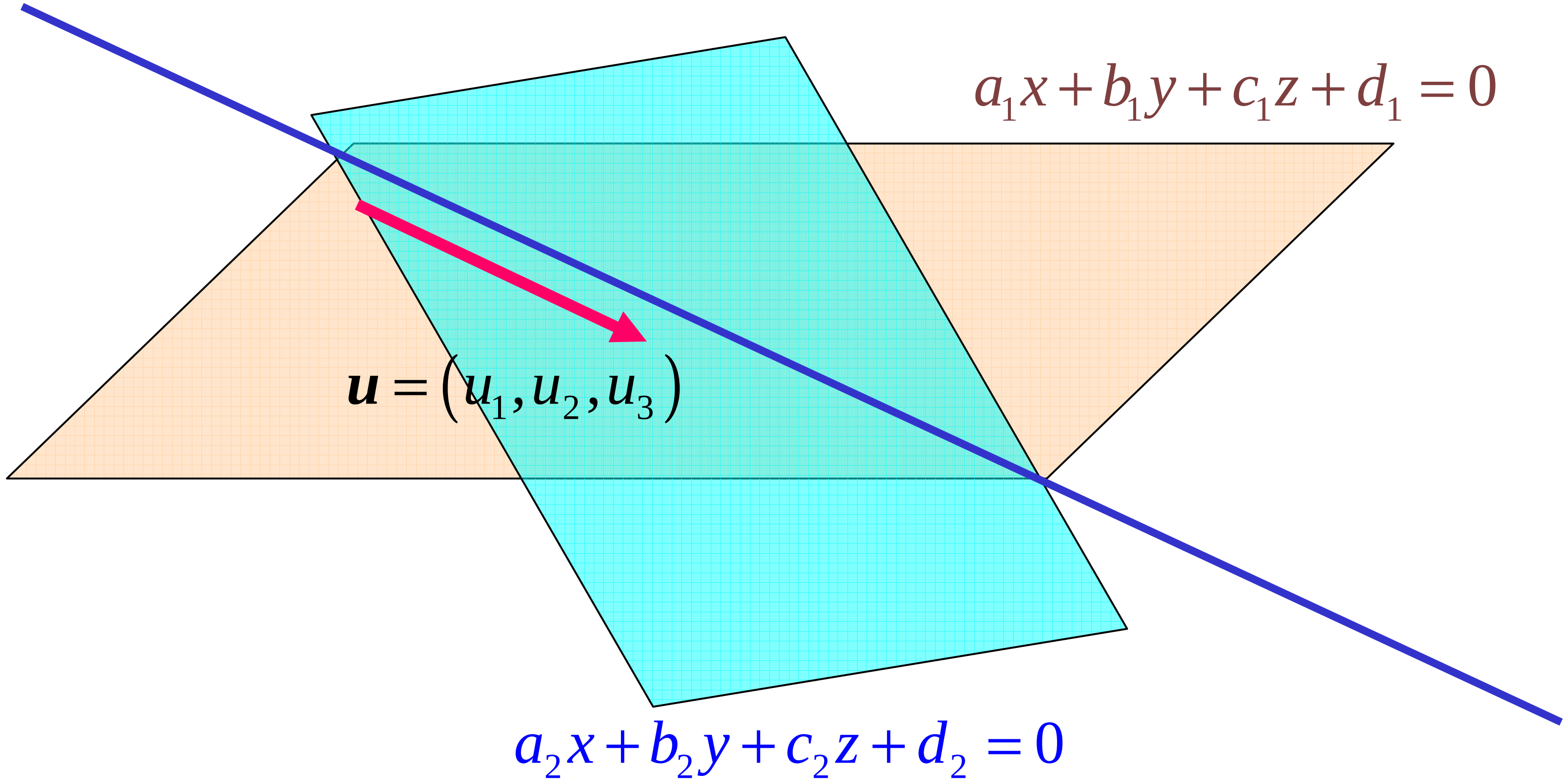
Given a general equation of a plane

$$ax + by + cz + d = 0$$



$$\boldsymbol{u} = k(a, b, c)$$

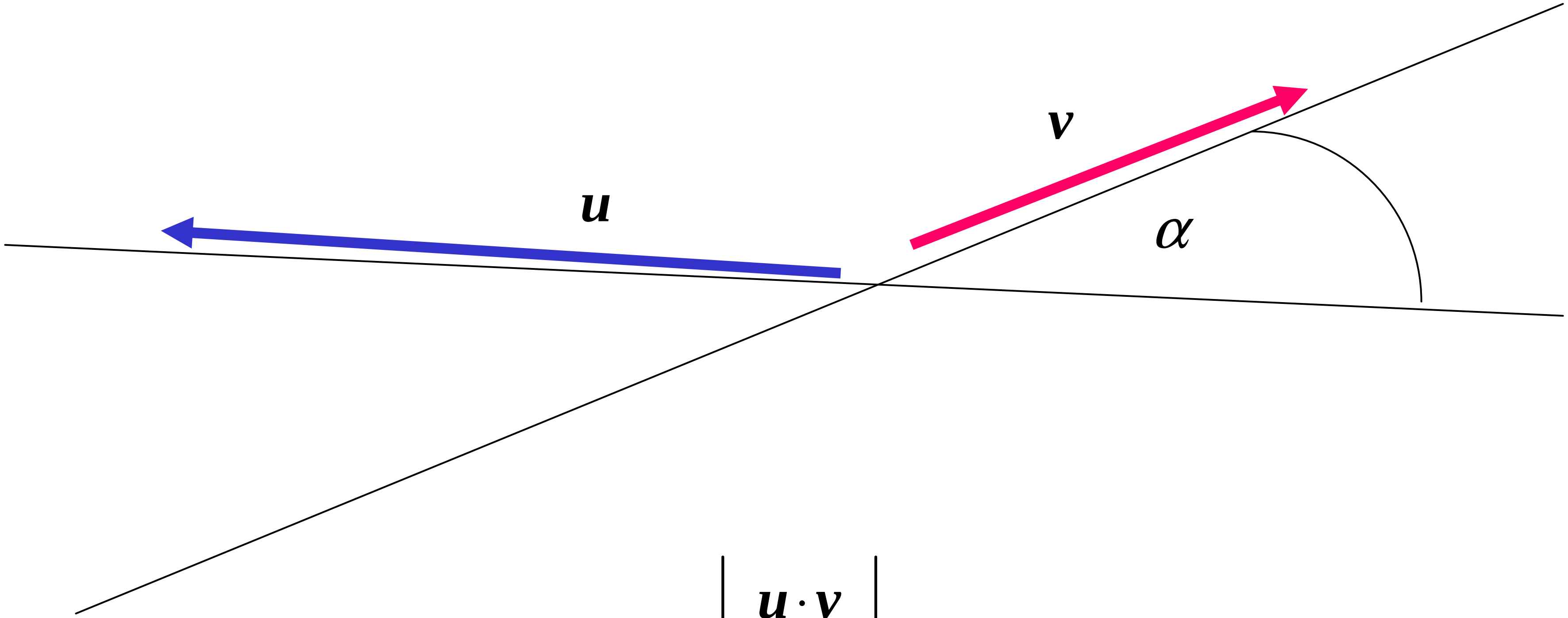
how can we determine a vector $\boldsymbol{u}$ perpendicular to that plane ?



$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = u_1\boldsymbol{i}+u_2\boldsymbol{j}+u_3\boldsymbol{k}$$

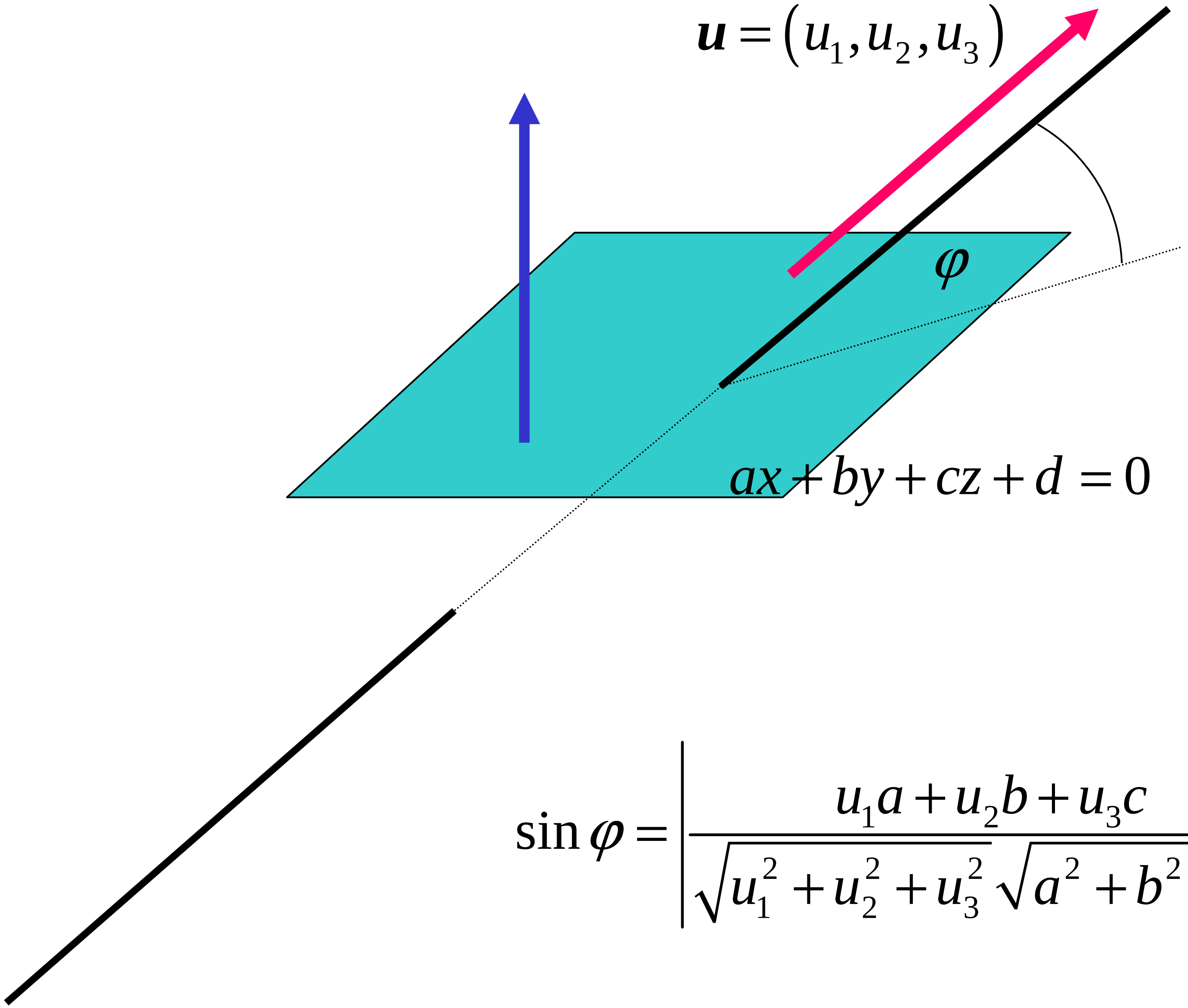
Finding the coordinates of a vector ***u*** parallel to the straight line given by two general equations of planes

Angle between two straight lines



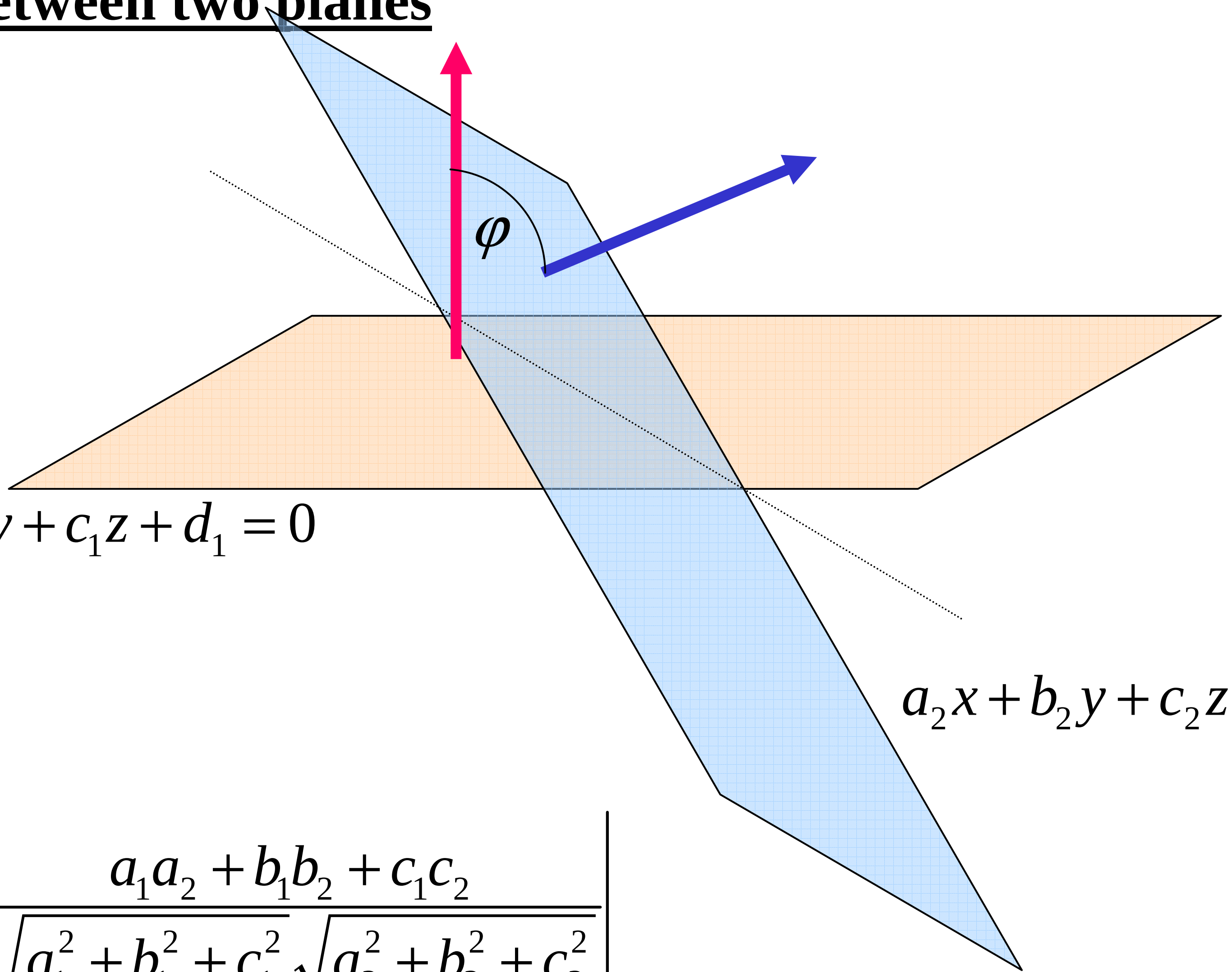
$$\cos\varphi = \left|\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}\right|$$

Angle between a plane and a straight line



$$\sin\varphi = \left| \frac{u_1 a + u_2 b + u_3 c}{\sqrt{u_1^2 + u_2^2 + u_3^2}\sqrt{a^2 + b^2 + c^2}} \right|$$

Angle between two planes

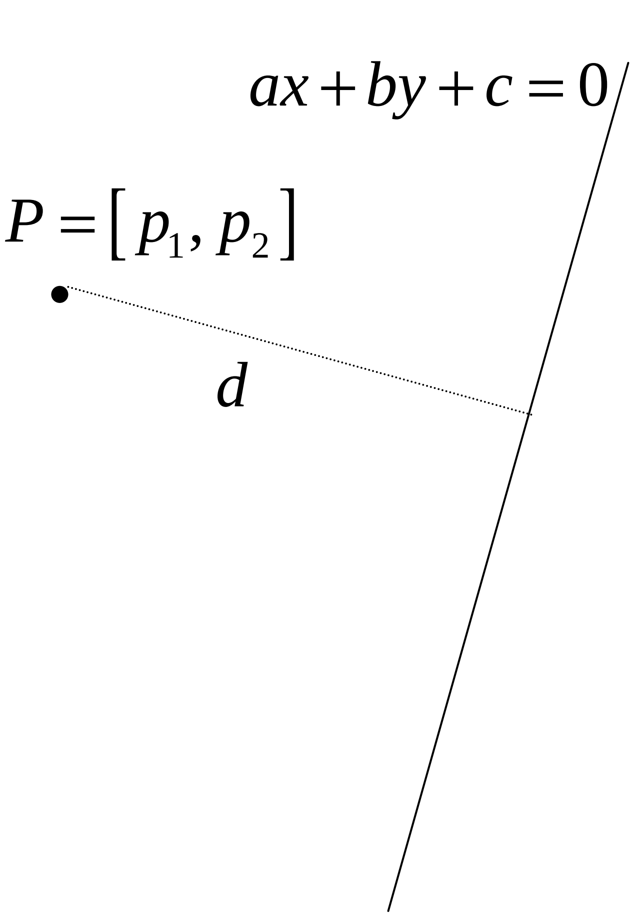


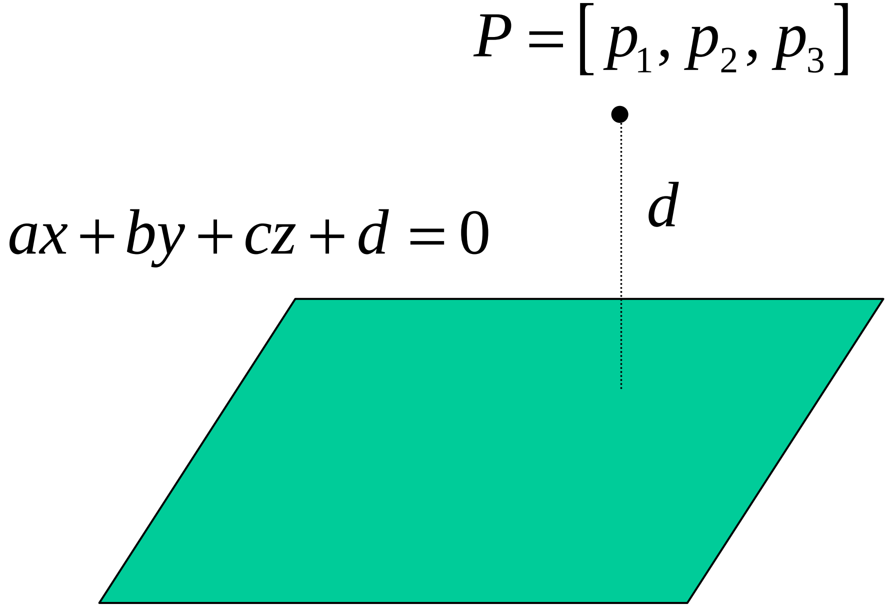
$$a_1 x + b_1 y + c_1 z + d_1 = 0$$

$$a_2 x + b_2 y + c_2 z + d_2 = 0$$

$$\cos\varphi = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

The distance of a point from a straight line or a plane

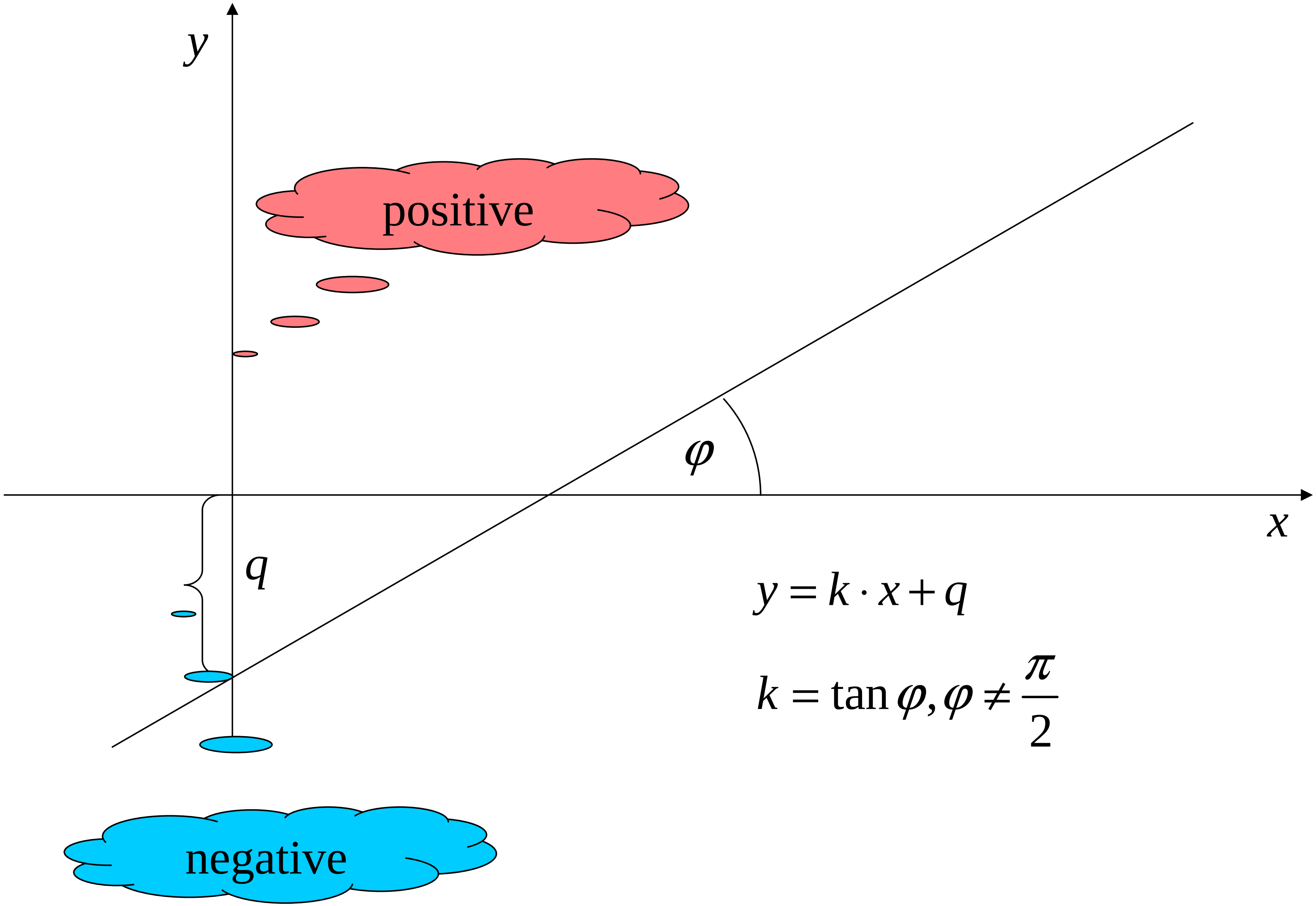




$$d = \frac{|ap_1 + bp_2 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|ap_1 + bp_2 + cp_3 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

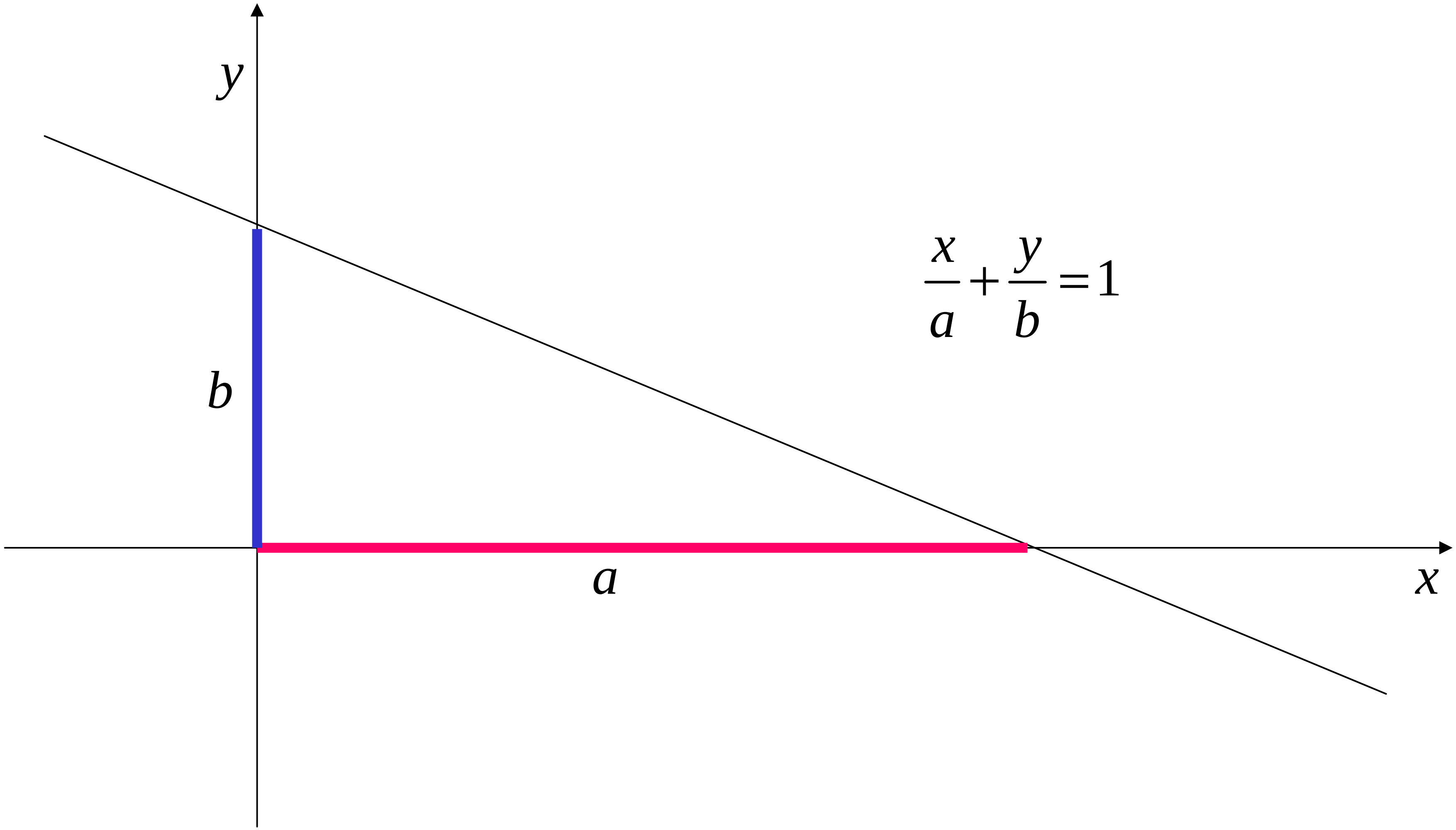
Special type equation of a straight line (1)



$$y = k \cdot x + q$$

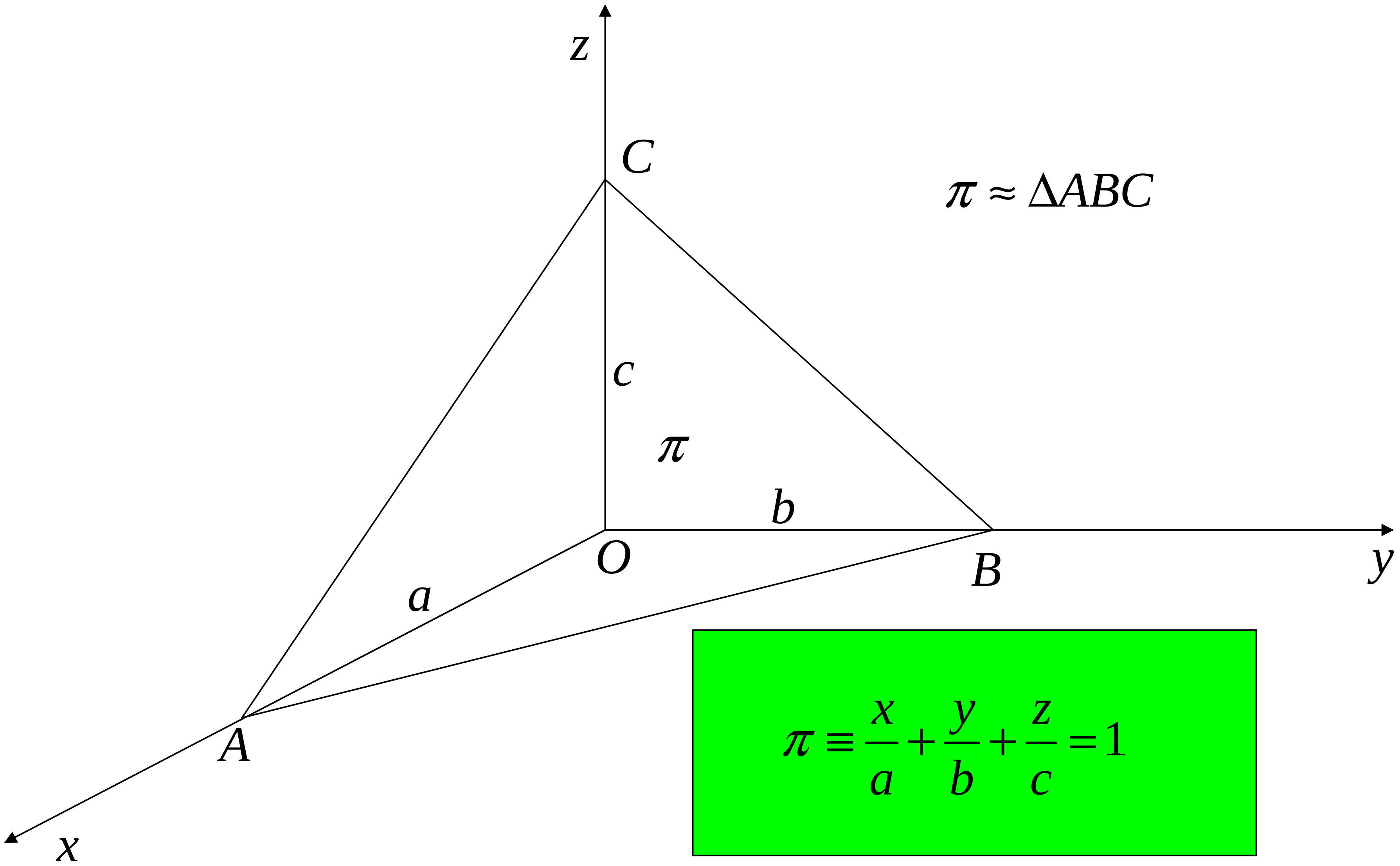
$$k = \tan\varphi, \varphi \neq \frac{\pi}{2}$$

Special type equation of a straight line (2)



$$\frac{x}{a}+\frac{y}{b}=1$$

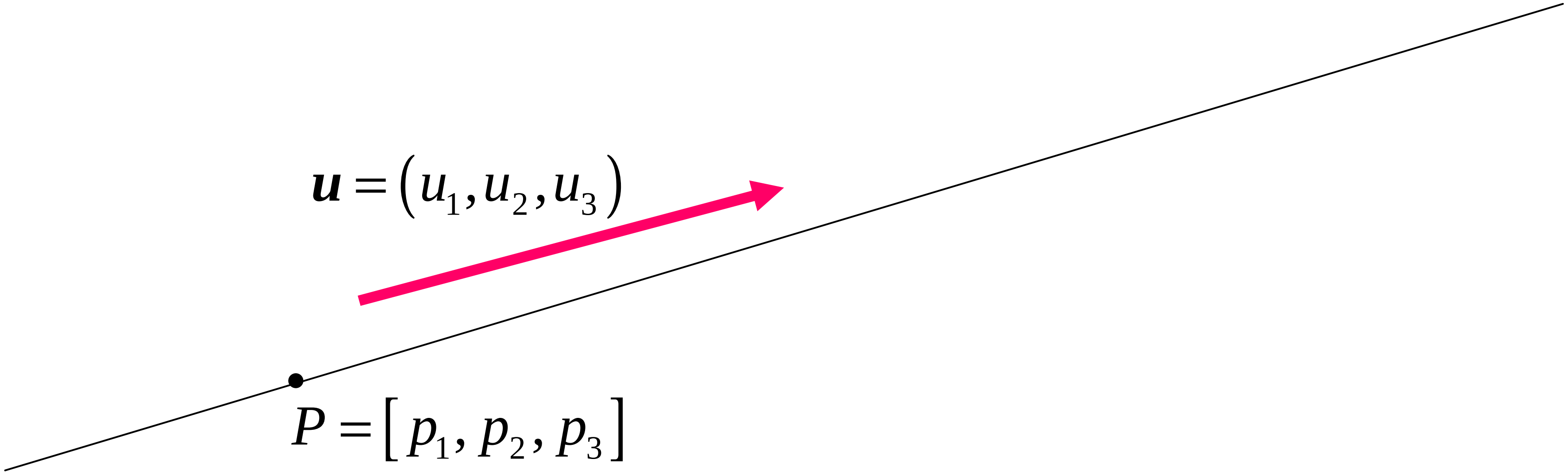
Special type equation of a plane



$$\pi \approx \Delta ABC$$

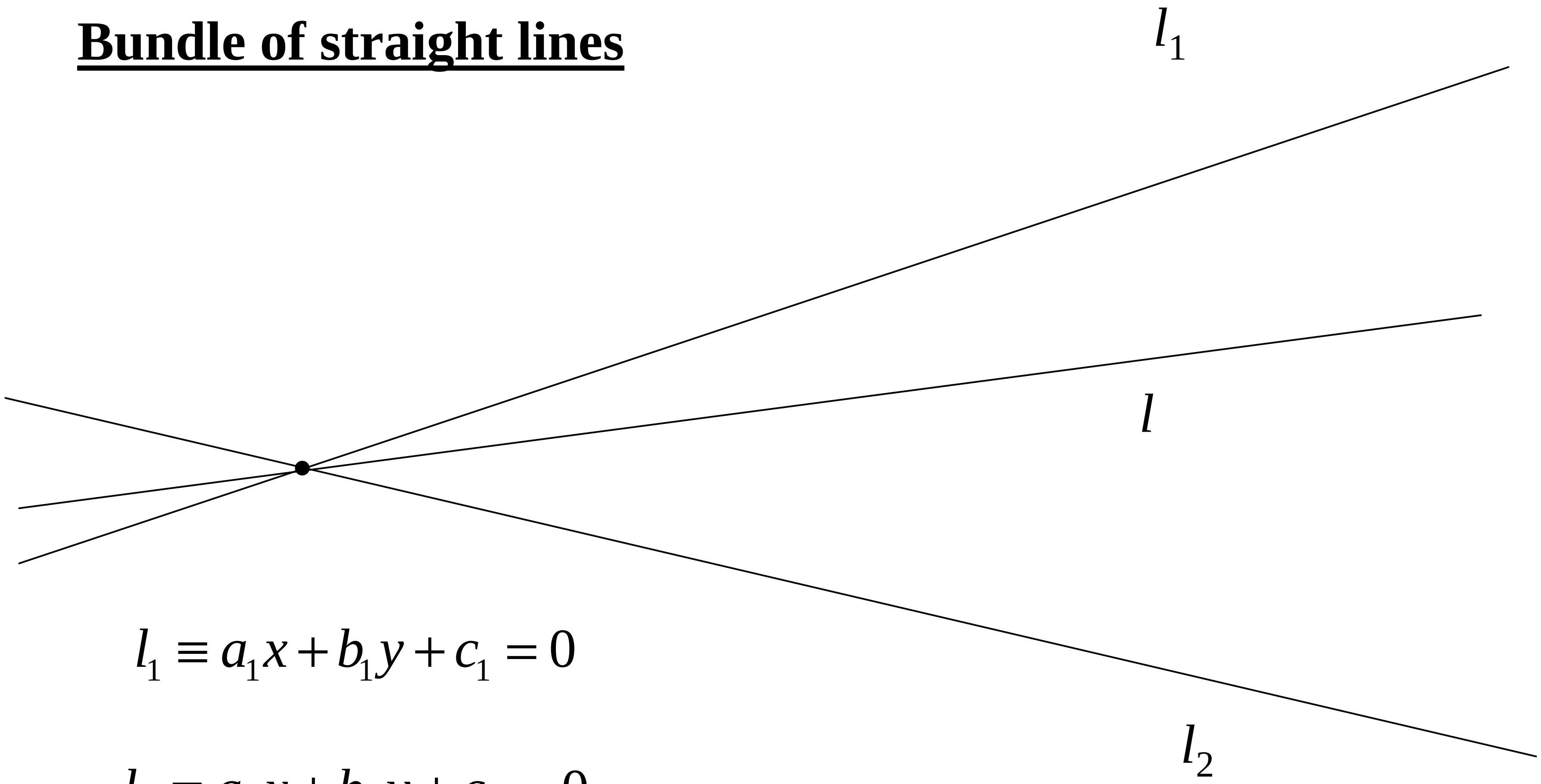
$$\pi \equiv \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Canonical equations of a straight line in E_3



$$\frac{x - p_1}{u_1} = \frac{y - p_2}{u_2} = \frac{z - p_3}{u_3}$$

<u>**Bundle of straight lines**</u>

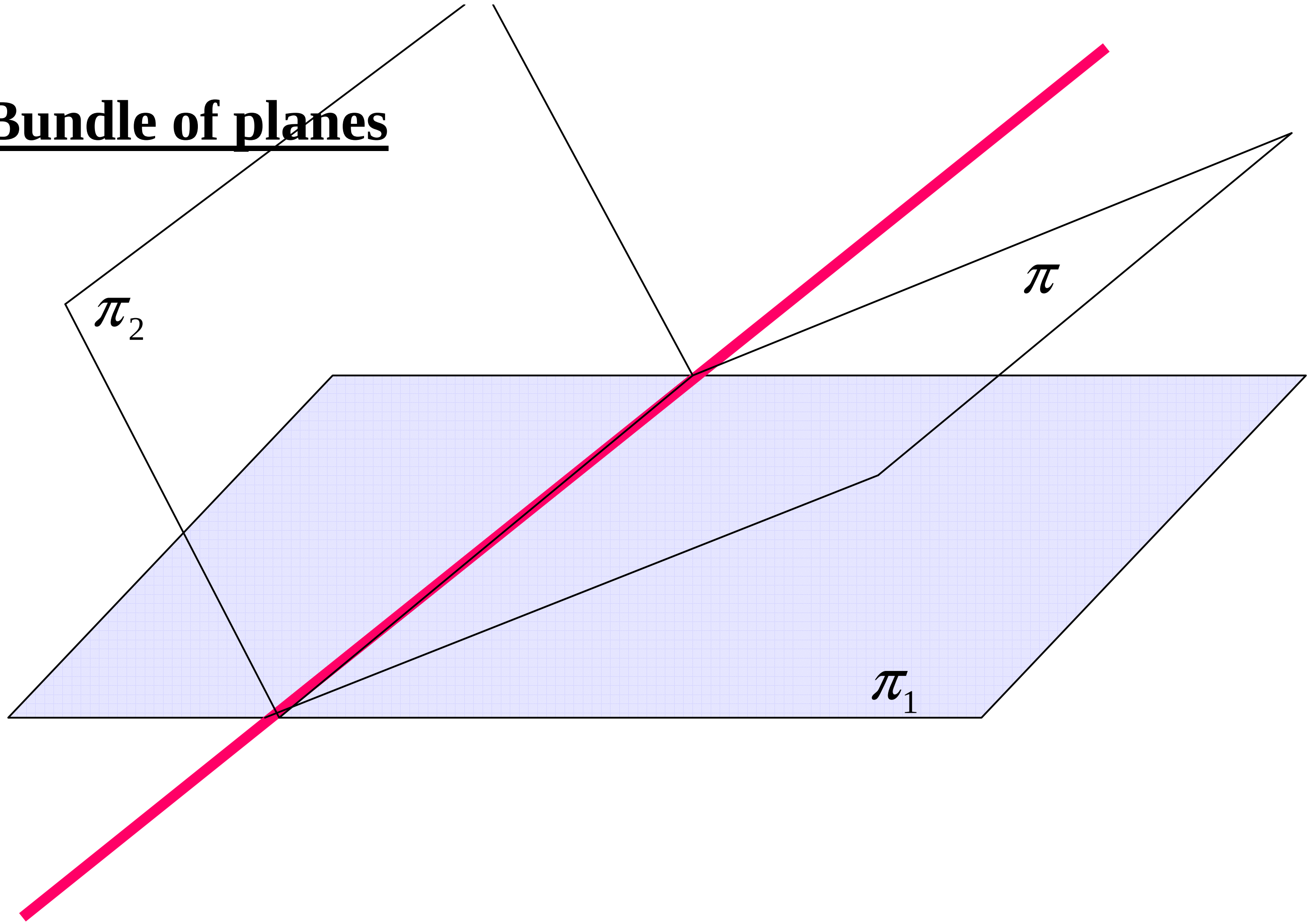


$$l_1 \equiv a_1 x + b_1 y + c_1 = 0$$

$$l_2 \equiv a_2 x + b_2 y + c_2 = 0$$

$$l \equiv \lambda_1 \left(a_1 x + b_1 y + c_1 + d_1 \right) + \lambda_2 \left(a_2 x + b_2 y + c_2 + d_2 \right) = 0$$

Bundle of planes

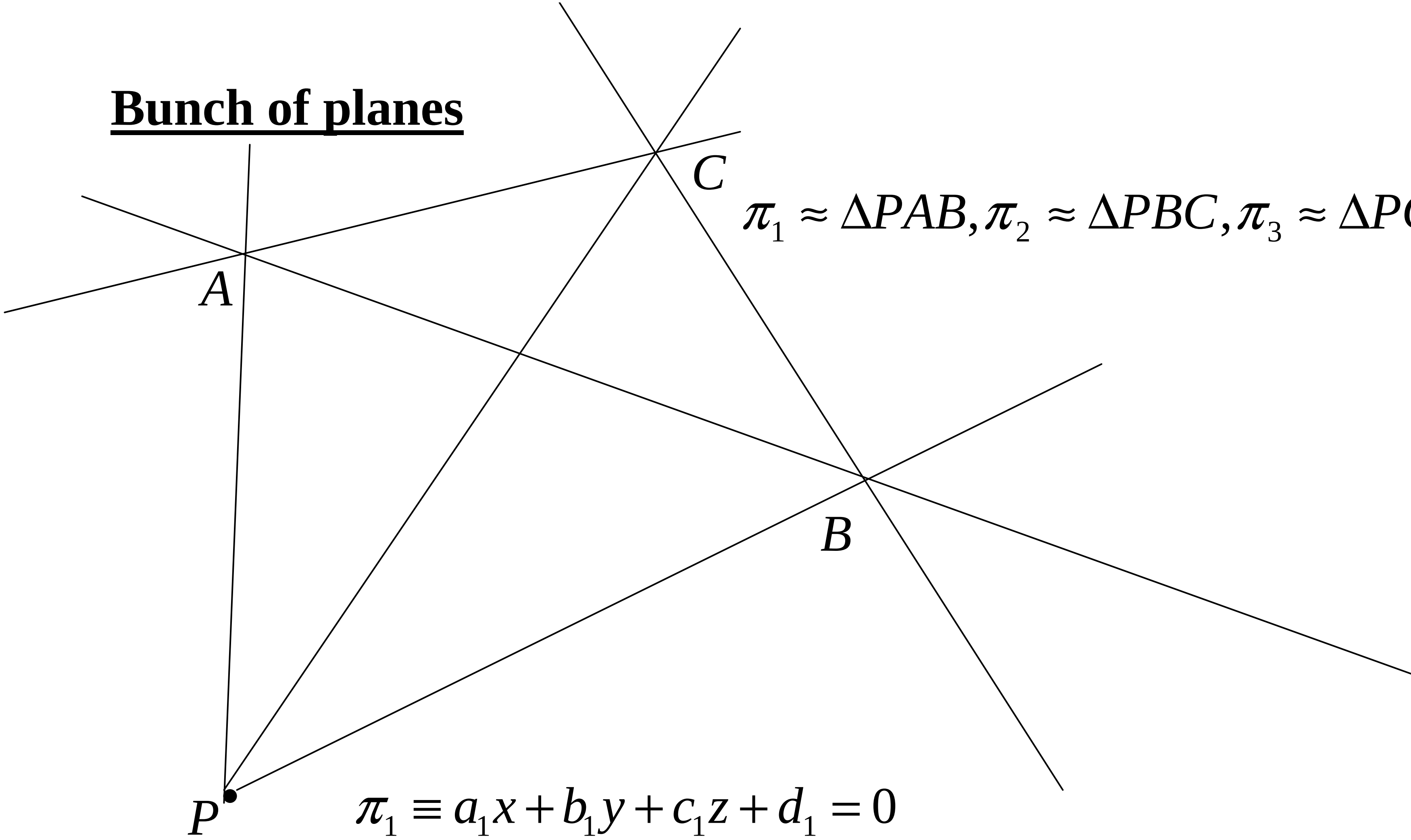


$$\pi_1 \equiv a_1x + b_1y + c_1z + d_1 = 0$$

$$\pi_2 \equiv a_2x + b_2y + c_2z + d_2 = 0$$

$$\pi \equiv \lambda_1\left(a_1x + b_1y + c_1z + d_1\right) + \lambda_2\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Bunch of planes



$$\pi_1 \approx \Delta PAB, \pi_2 \approx \Delta PBC, \pi_3 \approx \Delta PCA$$

$$\pi_1 \equiv a_1 x + b_1 y + c_1 z + d_1 = 0$$

$$\pi_2 \equiv a_2 x + b_2 y + c_2 z + d_2 = 0$$

$$\pi_3 \equiv a_3 x + b_3 y + c_3 z + d_3 = 0$$

$$\pi \equiv \lambda_1 (\pi_1) + \lambda_2 (\pi_2) + \lambda_3 (\pi_3) = 0$$