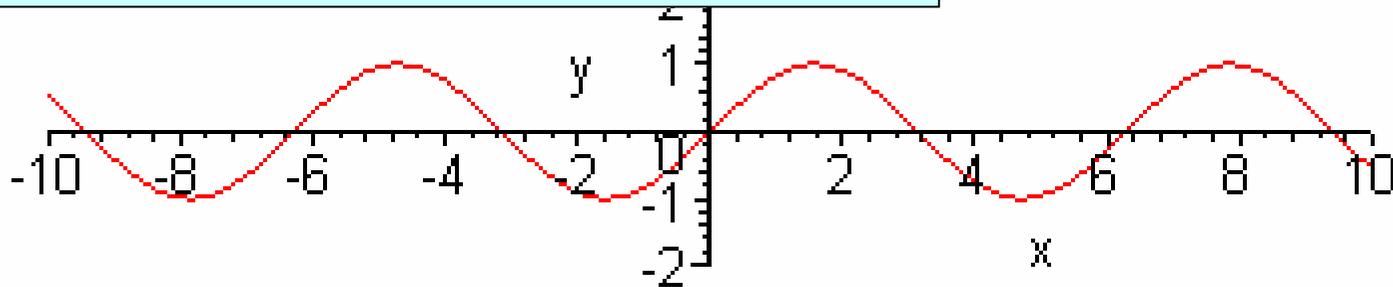
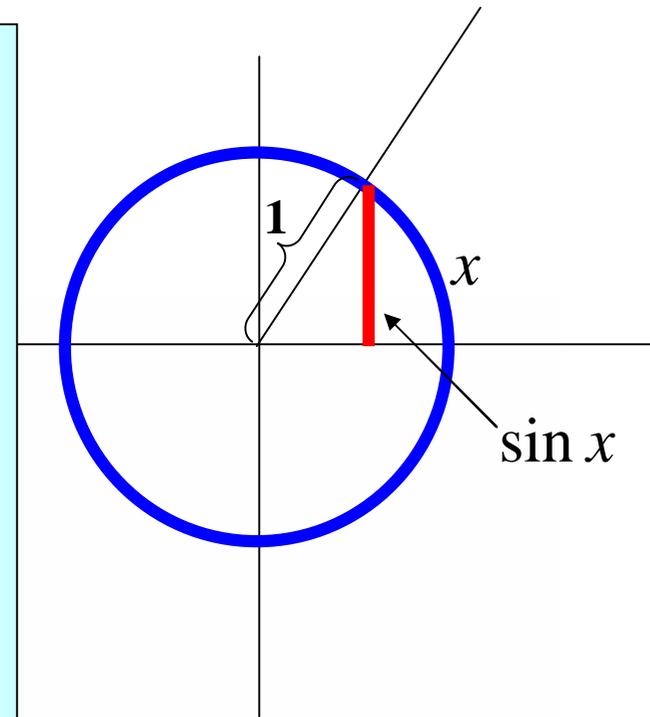


## The sine

The sine function written as  $\sin(x)$  or just  $\sin x$  is geometrically defined as demonstrated by the figure to the right where  $x$  is the length of the arc.

Its basic domain is  $(0, 2\pi)$ , which is extended over the entire  $x$ -axis so that the function becomes periodic.

Its range is  $(-1, 1)$ .



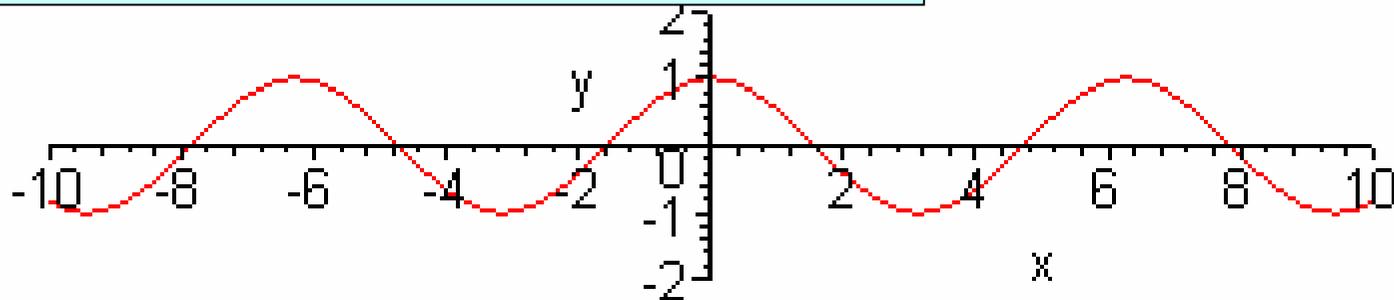
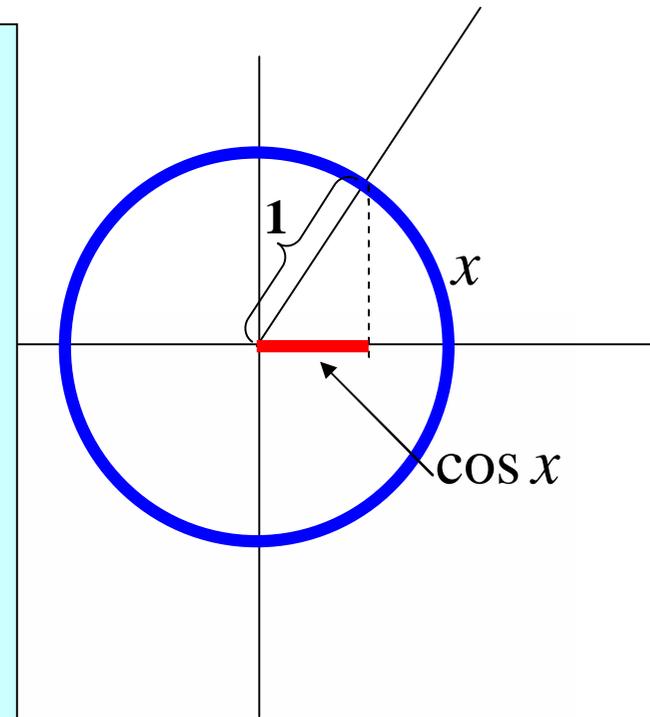
$\sin x$  is an odd, periodic function with  $2\pi$  as the least period

## The cosine

The cosine function written as  $\cos(x)$  or just  $\cos x$  is geometrically defined as demonstrated by the figure to the right where  $x$  is the length of the arc

Its basic domain is  $(0, 2\pi)$ , which is extended over the entire  $x$ -axis so that the function becomes periodic.

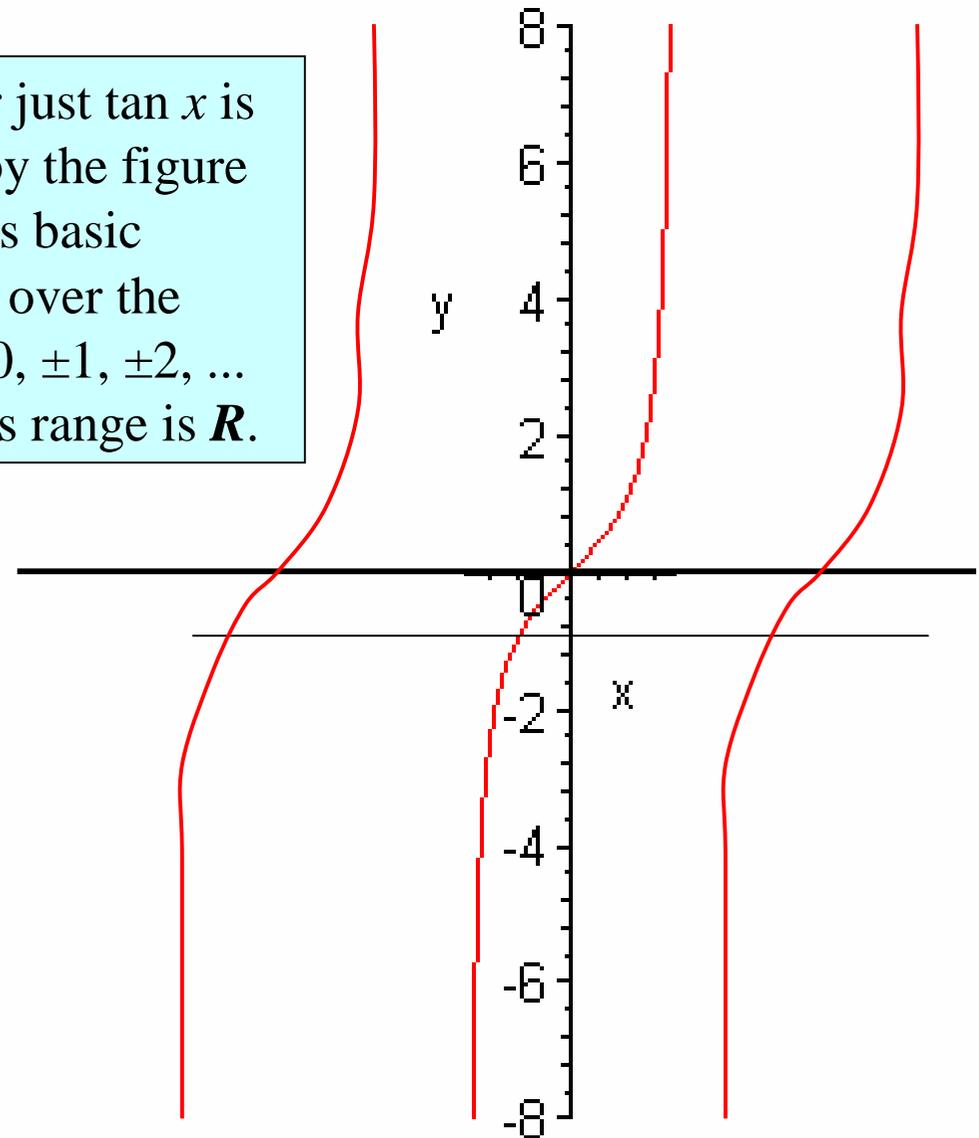
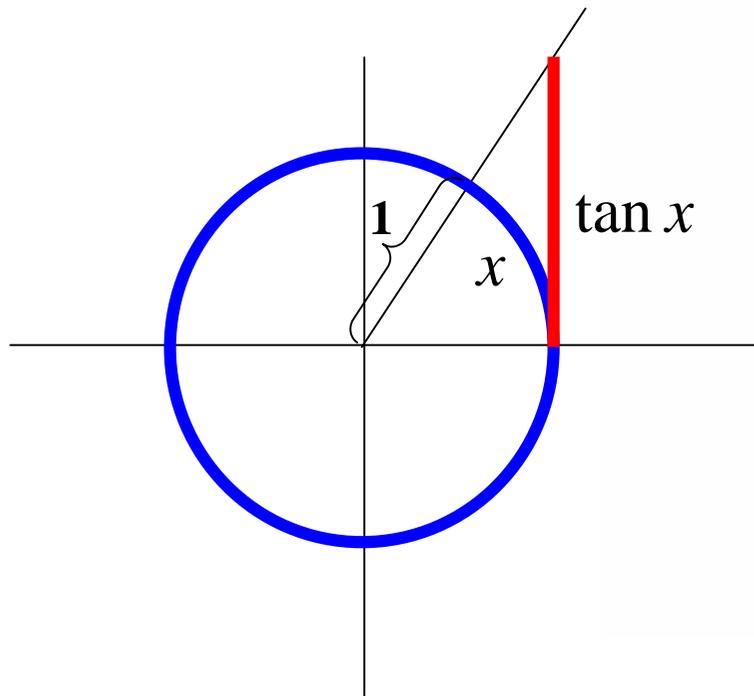
Its range is  $(-1,1)$ .



$\cos x$  is an even, periodic function with  $2\pi$  as the least period

## The tangent

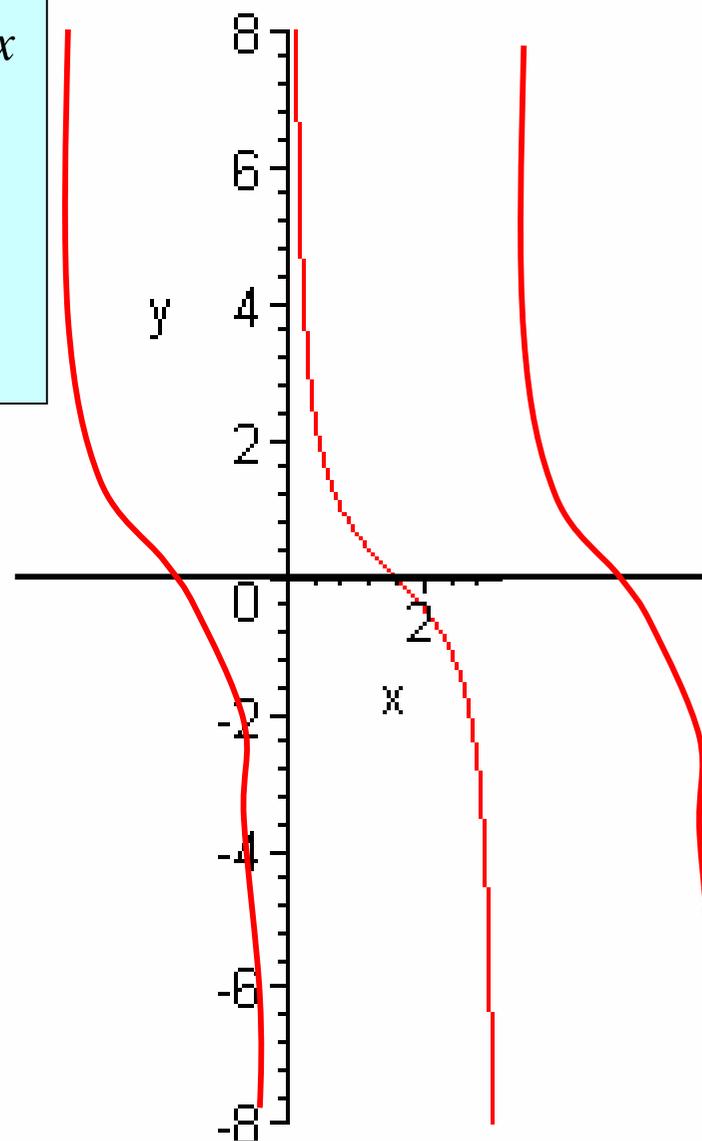
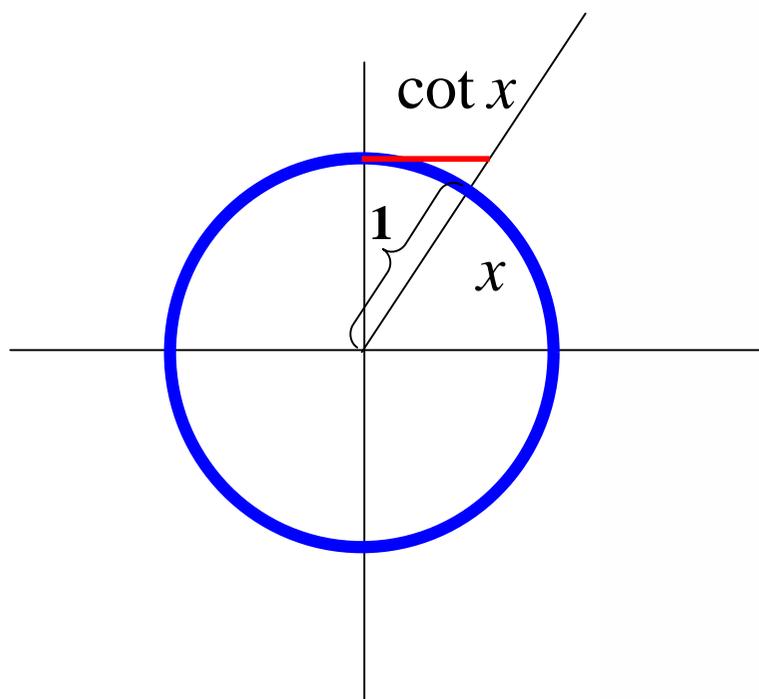
The tangent function written as  $\tan(x)$  or just  $\tan x$  is geometrically defined as demonstrated by the figure below where  $x$  is the length of the arc. Its basic domain is  $(-\pi/2, \pi/2)$ , which is extended over the entire  $x$ -axis except at  $x = \pi/2 + k\pi, k=0, \pm 1, \pm 2, \dots$  so that the function becomes periodic. Its range is  $\mathbf{R}$ .



$\tan x$  is an odd, periodic function with  $\pi$  as the least period

## The cotangent

The cotangent function written as  $\cot(x)$  or just  $\cot x$  is geometrically defined as demonstrated by the figure below where  $x$  is the length of the arc. Its basic domain is  $(0, \pi)$ , which is extended over the entire  $x$ -axis except at  $k\pi, x = 0, \pm 1, \pm 2, \dots$  so that the function becomes periodic. Its range is  $\mathbf{R}$ .



$\cot x$  is an odd, periodic function with  $\pi$  as the least period

When the argument of a trigonometric function is viewed as an angle, its value may be given in degrees where  $\pi = 180^\circ$

Here are some useful values of trigonometric functions:

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin x$	0	$1/2$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$1/2$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\infty$
$\cot x$	$\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

## Some useful trigonometric formulas:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \sqrt{1 - \cos^2 x}$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

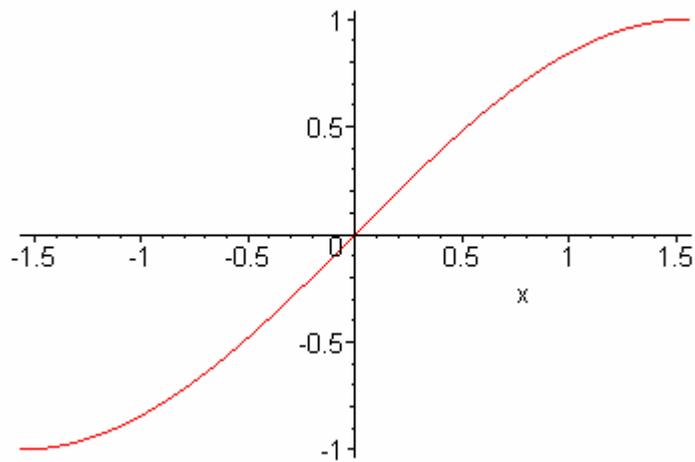
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

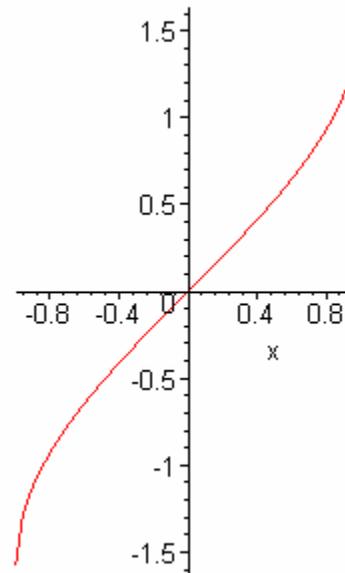
## The inverse sine

$f(x) = \arcsin x$  Sometimes called the *arc sine* function

When inverting the sine function, we have to restrict ourselves to the domain  $(-\pi/2, \pi/2]$  where  $\sin(x)$  is one-to-one. Then  $R(f) = (-\pi/2, \pi/2]$  and  $D(f) = [-1, 1]$ .



$$y = \sin(x)$$



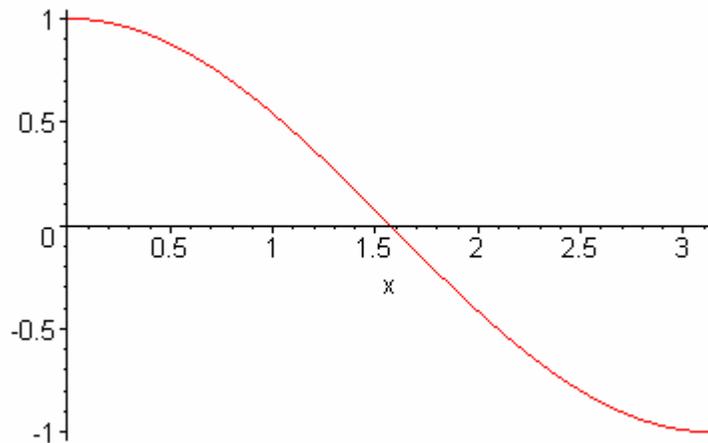
$$y = \arcsin x$$

## The inverse cosine

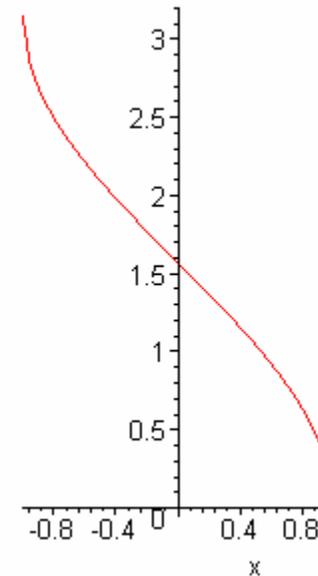
$f(x) = \arccos x$  Sometimes called the *arc cosine* function

When inverting the cosine function, we have to restrict ourselves to the domain  $(0, p]$  where  $\cos(x)$  is one-to-one.

Then  $R(f) = (0, p]$  and  $D(f) = [-1, 1]$ .



$$y = \cos(x)$$



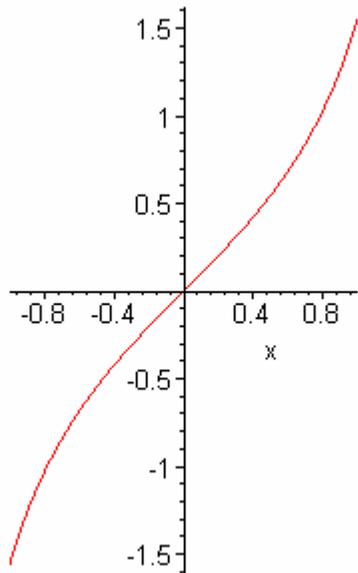
$$y = \arccos x$$

## The inverse tangent

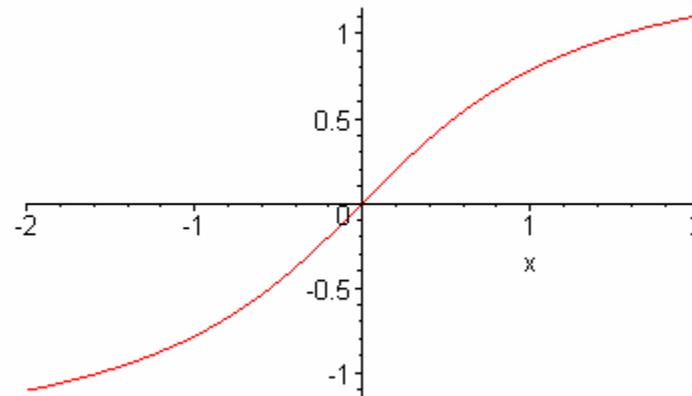
$f(x) = \arctan x$  Sometimes called the *arc tangent* function

When inverting the tangent function, we have to restrict ourselves to the domain  $(-\pi/2; \pi/2)$  where  $\tan(x)$  is one-to-one.

Then  $R(f) = (-\pi/2; \pi/2)$  and  $D(f) = (-\infty; \infty)$



$$y = \tan(x)$$



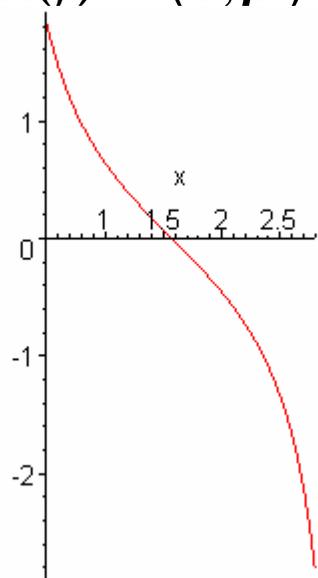
$$y = \arctan x$$

## The inverse cotangent

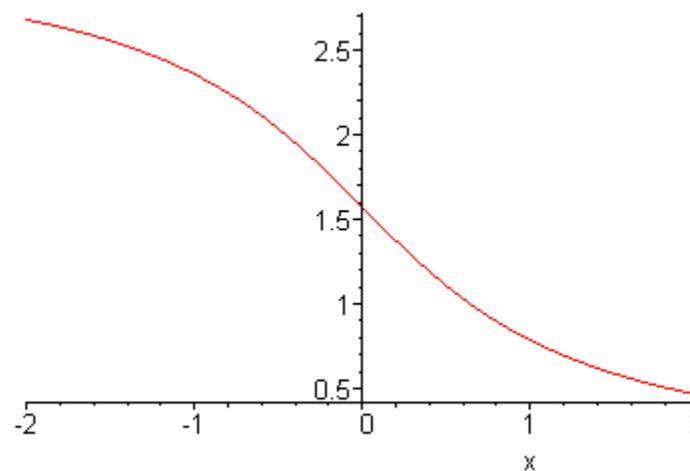
$f(x) = \text{arc cot } x$  Sometimes called the *arc cotangent* function

When inverting the cotangent function, we have to restrict ourselves to the domain  $(0; p)$  where  $\tan(x)$  is one-to-one.

Then  $R(f) = (0; p)$  and  $D(f) = (-\infty; \infty)$



$$y = \cot(x)$$



$$y = \text{arc cot } x$$