Exponential functions

$$f(x) = a^x, \quad a > 0$$

For $n \in N$ we put $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

Next we can extend the domain of $f(x)$ to $\boldsymbol{Z}$ by putting

$$a^{-n} = \frac{1}{a^n}$$

and $a^0 = 1$

to satisfy the condition $a^n \cdot a^m = a^{m+n}$

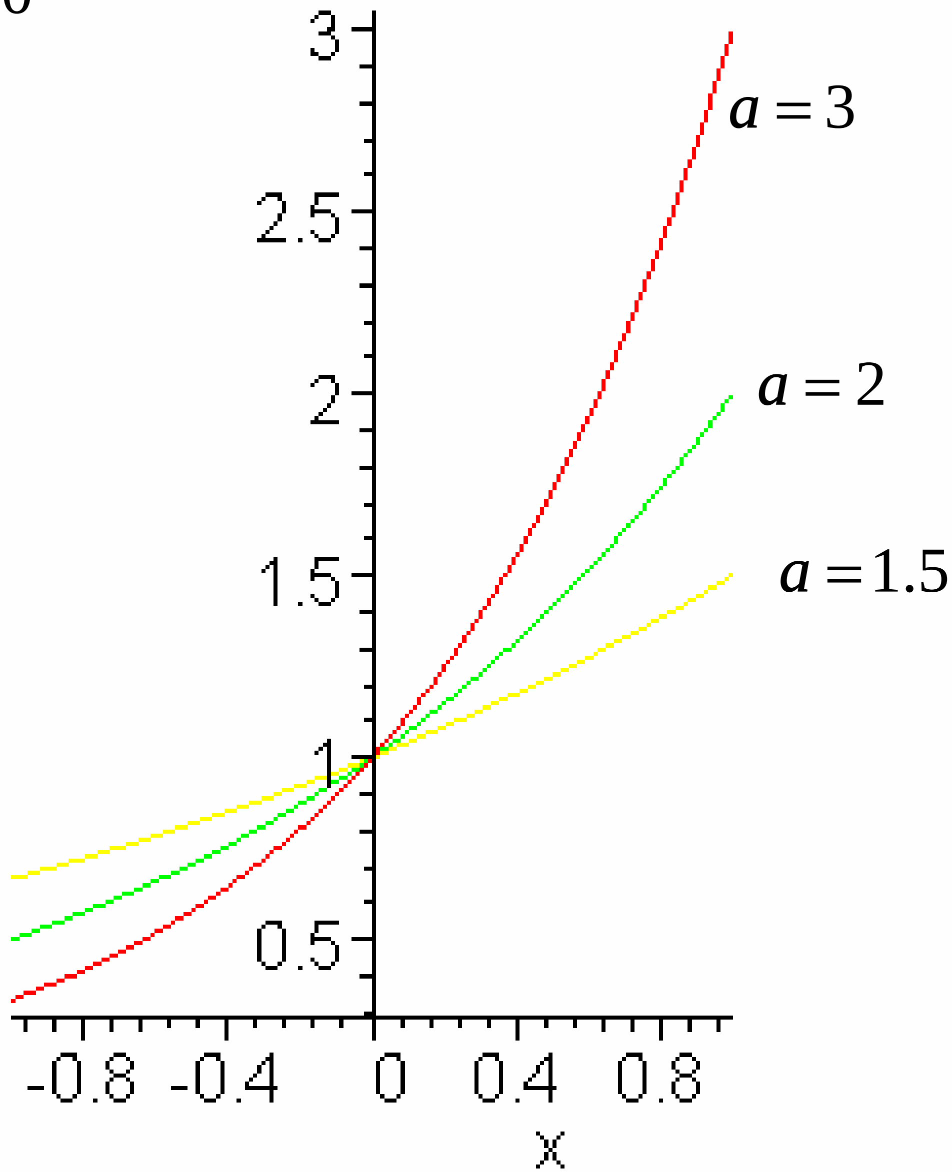
Considering that $\left(\sqrt[n]{a}\right)^n = 1$, we add $a^{\frac{1}{n}} = \sqrt[n]{a}$

Thus for every rational $r = \dfrac{p}{q}$ we can write $a^r = \left(\sqrt[q]{a}\right)^p$

since $\left(a^p\right)^q = a^{pq}$

We could prove that the function $f(x) = a^x$ can even be defined for irrational real numbers by "filling up the holes in a "smooth way".

$f(x) = a^x, a > 0$



$$f\left(x\right)=e^{x}$$ sometimes written as $$f\left(x\right)=\exp(x)$$

$$e=2{,}718\;281\,828\;459\;045\;\mathbf{K}$$

$$\left(1+\frac{1}{1}\right)^{1}\rightarrow\left(1+\frac{1}{2}\right)^{2}\rightarrow\left(1+\frac{1}{2}\right)^{2}\rightarrow\mathbf{L}\rightarrow\left(1+\frac{1}{n}\right)^{n}\rightarrow\mathbf{L}\quad e$$

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\mathbf{L}$$

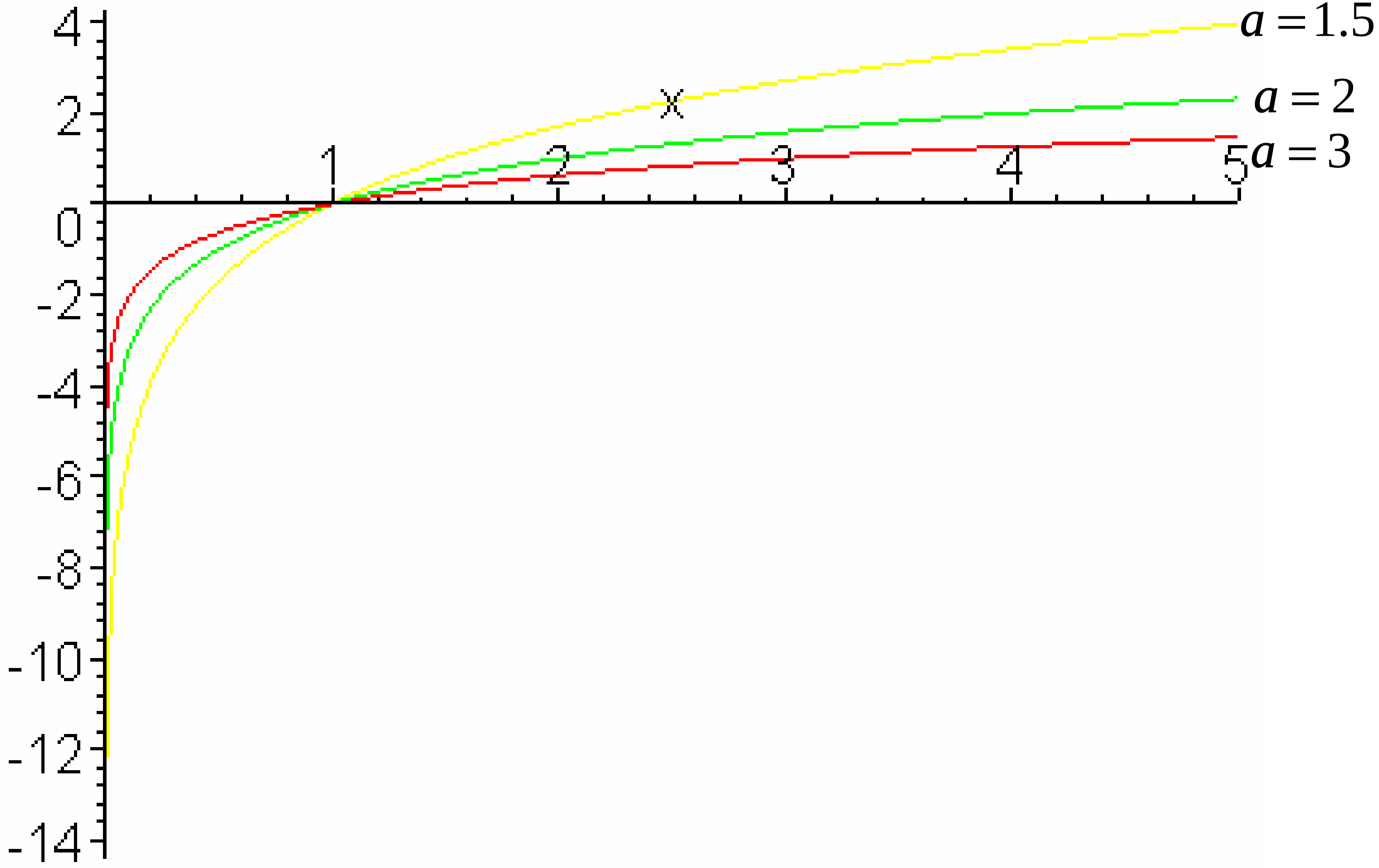
Logarithmic functions

For each exponential function $f(x) = a^x$ we can define its inverse since $f(x)$ is a one-to-one mapping

$$f^{-1}(x) = \log_a x$$

The number $\log_a x$ is called the logarithm of x to the base a

$$f(x) = \log_a x$$



The logarithm of x to the base e is called

the natural logarithm of x.

$$\log_e x = \ln x$$