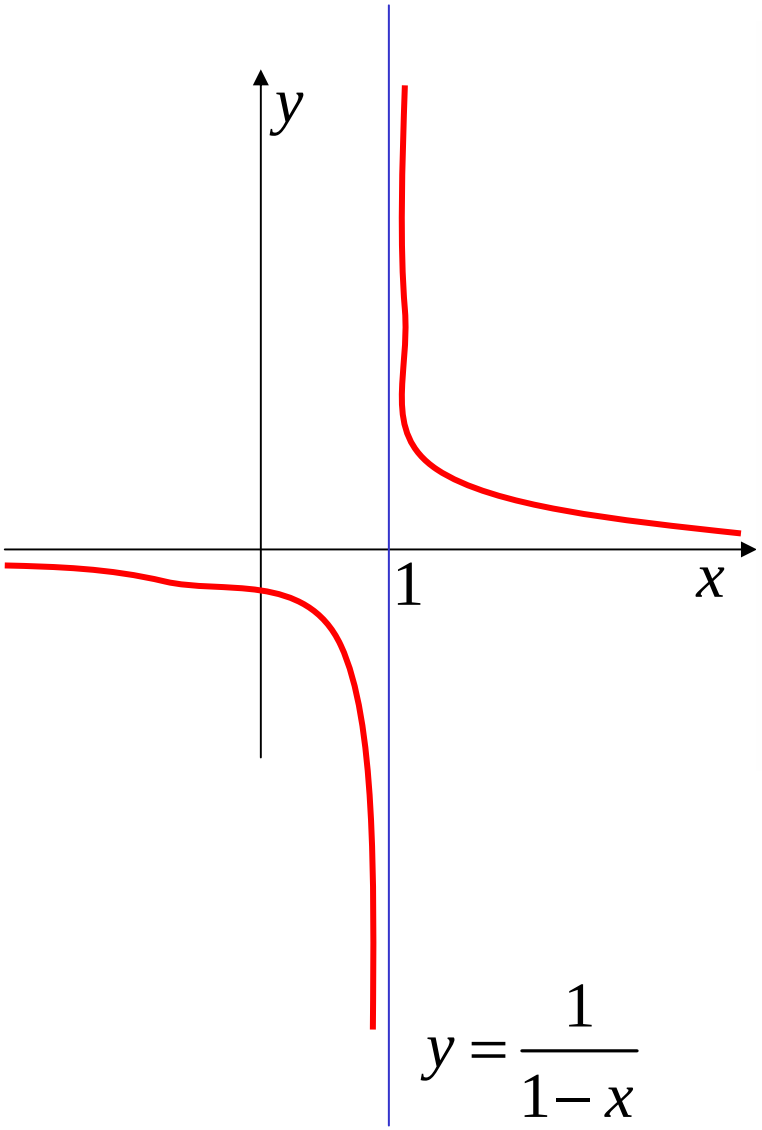
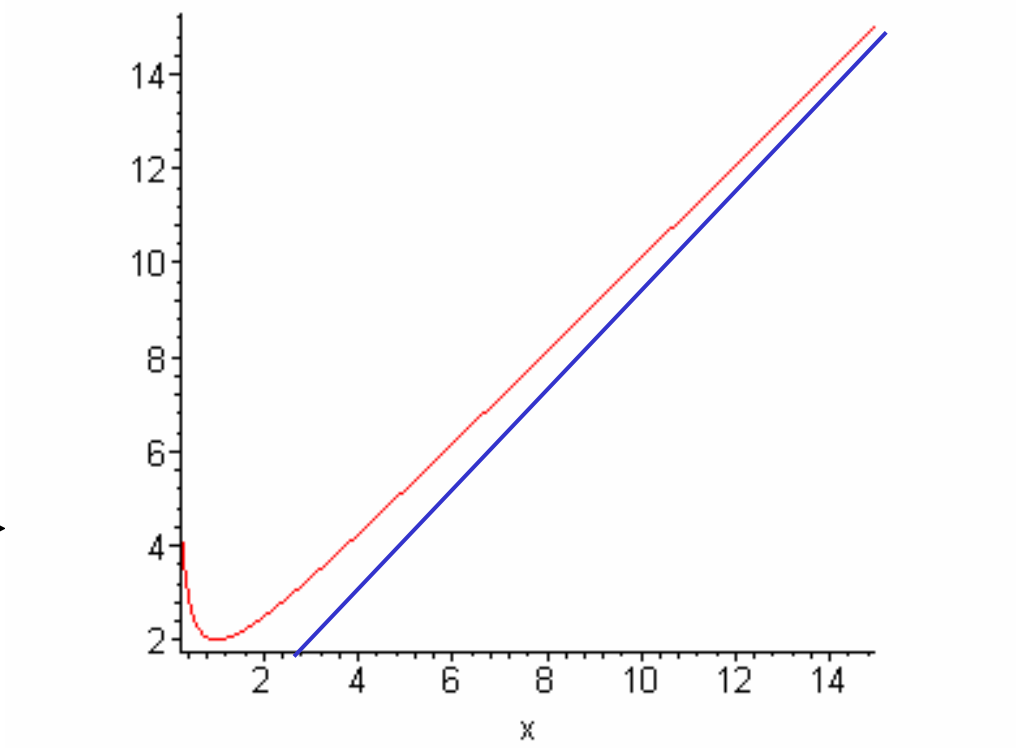
Types of asymptotes



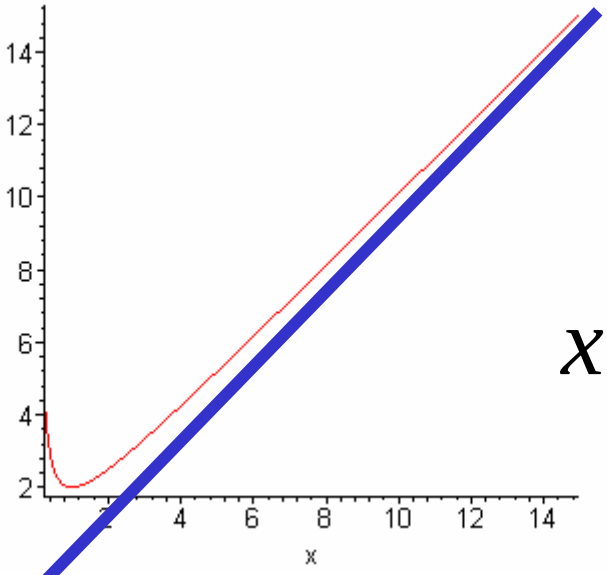
$$y = \frac{1}{1-x}$$



$$y = \frac{x^2+1}{x}$$

Finding the equation of asymptote with a slope

$y = f(x)$



$x \to \infty$

$y = Ax + B$

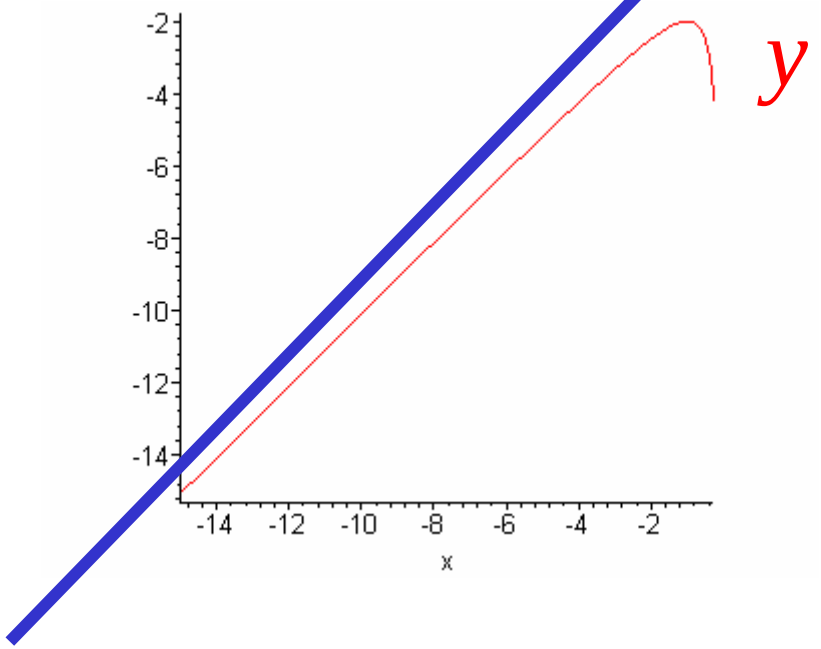
$$A = \lim_{x \to \infty} \frac{f(x)}{x}$$

$$B = \lim_{x \to \infty} \left(f(x) - Ax \right)$$

Finding the equation of asymptote with a slope

$$y = Ax + B$$

$$x \to -\infty$$



$$y = f(x)$$

$$A = \lim_{x \to -\infty} \frac{f(x)}{x}$$

$$B = \lim_{x \to -\infty} \left(f(x) - Ax \right)$$

Example

$$y=\frac{2x^2+1}{x}$$

$$A=\lim_{x\to\infty}\frac{\frac{2x^2+1}{x}}{x}=\lim_{x\to\infty}\frac{2x^2+1}{x^2}=2$$

$$B=\lim_{x\to\infty}\frac{2x^2+1}{x}-2\cdot x=\lim_{x\to\infty}\frac{2x^2+1-2x^2}{x}=\lim_{x\to\infty}\frac{1}{x}=0$$

$$A=\lim_{x\to-\infty}\frac{\frac{2x^2+1}{x}}{x}=\lim_{x\to-\infty}\frac{2x^2+1}{x^2}=2$$

$$B=\lim_{x\to-\infty}\frac{2x^2+1}{x}-2\cdot x=\lim_{x\to-\infty}\frac{2x^2+1-2x^2}{x}=\lim_{x\to-\infty}\frac{1}{x}=0$$

Sketching the graph of a function $y = f(x)$

- determine the domain of f, determine asymptotes without slope
- find intervals in which f is positive and negative
- calculate f' and its domain
- find intervals in which f' is positive and negative
- calculate f'' and its domain
- find intervals in which f'' is positive and negative
- find asymptotes with a slope if any
- sketch the graph of f