Calculating definite integrals by parts

$$\int_a^b f'(x)\,g(x)\,dx = \left[g(x)\,f(x)\right]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

Example

$$\int_1^e \ln x \, dx = \left| \begin{array}{ll} f'(x) = 1 & f(x) = x \\ g(x) = \ln x & g'(x) = \dfrac{1}{x} \end{array} \right| = \left[x \ln x \right]_1^e - \int_1^e 1 \, dx =$$

$$= \left[x \ln x \right]_1^e - \left[x \right]_1^e = e - 0 - e + 1 = 1$$

Calculating definite integrals using substitutions

If a function f (x) is continuous for $a \le x \le b$ and $x = \varphi(t)$

is a function continuous with its first derivative $\varphi'(t)$ on an interval

$\alpha \le t \le \beta$ where $a = \varphi(\alpha), b = \varphi(\beta)$ and $f\left[\varphi(t)\right]$ is

defined and continuous for $\alpha \le t \le \beta$, then

$$\int_a^b f(x)\,dx = \int_\alpha^\beta f\left[\varphi(t)\right]\varphi'(t)\,dt$$

<u>Example</u>

$$\int_0^a x^2\sqrt{a^2-x^2}\,dx = \left|\begin{matrix} x = a\sin t \\ dx = a\cos t\,dt \end{matrix}\right| = \int_0^{\pi/2} a^2\sin^2 t\sqrt{a^2-a^2\sin^2 t}\;a\cos t\,dt =$$

$$= a^4\int_0^{\pi/2}\sin^2 t\cos^2 t\,dt = \frac{a^4}{4}\int_0^{\pi/2}\sin^2 2t\,dt = \frac{a^4}{8}\int_0^{\pi/2}1-\cos 4t\,dt =$$

$$= \frac{a^4}{8}\left[t\right]_0^{\pi/2} - \frac{a}{32}\left[\sin 4t\right]_0^{\pi/2} = \frac{a^4\pi}{16} - 0 + 0 = \frac{a^4\pi}{16}$$

Could we calculate the integral

$$\int_0^2 \sqrt[3]{1 - x^2}\, dx$$

using the substitution

$$x = \cos t$$

?

Estimates of definite integrals

If $f(x) \le F(x)$ for $a \le x \le b$, then

$$\boxed{\int_a^b f(x)\,dx \le \int_a^b F(x)\,dx}$$

Estimates of definite integrals

If $f\left(x\right), \varphi\left(x\right)$ are continuous for $a \le x \le b$ and moreover $\varphi\left(x\right) \ge 0$ then

$$\boxed{m\int_a^b \varphi\left(x\right)\,dx \le \int_a^b f\left(x\right)\varphi\left(x\right)\,dx \le M\int_a^b \varphi\left(x\right)\,dx}$$

where $m \le f\left(x\right) \le M$ for $a \le x \le b$.

Estimates of definite integrals

If, in particular, we take $\varphi(x) \equiv 1 \quad a \leq x \leq b$, we get

$$\boxed{m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)}$$

where $m \leq f(x) \leq M$ for $a \leq x \leq b$.

Mean value theorem for definite integrals I

If $f(x), \varphi(x)$ are continuous for $a \le x \le b$ and moreover

$\varphi(x) \ge 0$ then

$$\boxed{\int_a^b f(x)\varphi(x)\,dx = f(c)\int_a^b \varphi(x)\,dx}$$

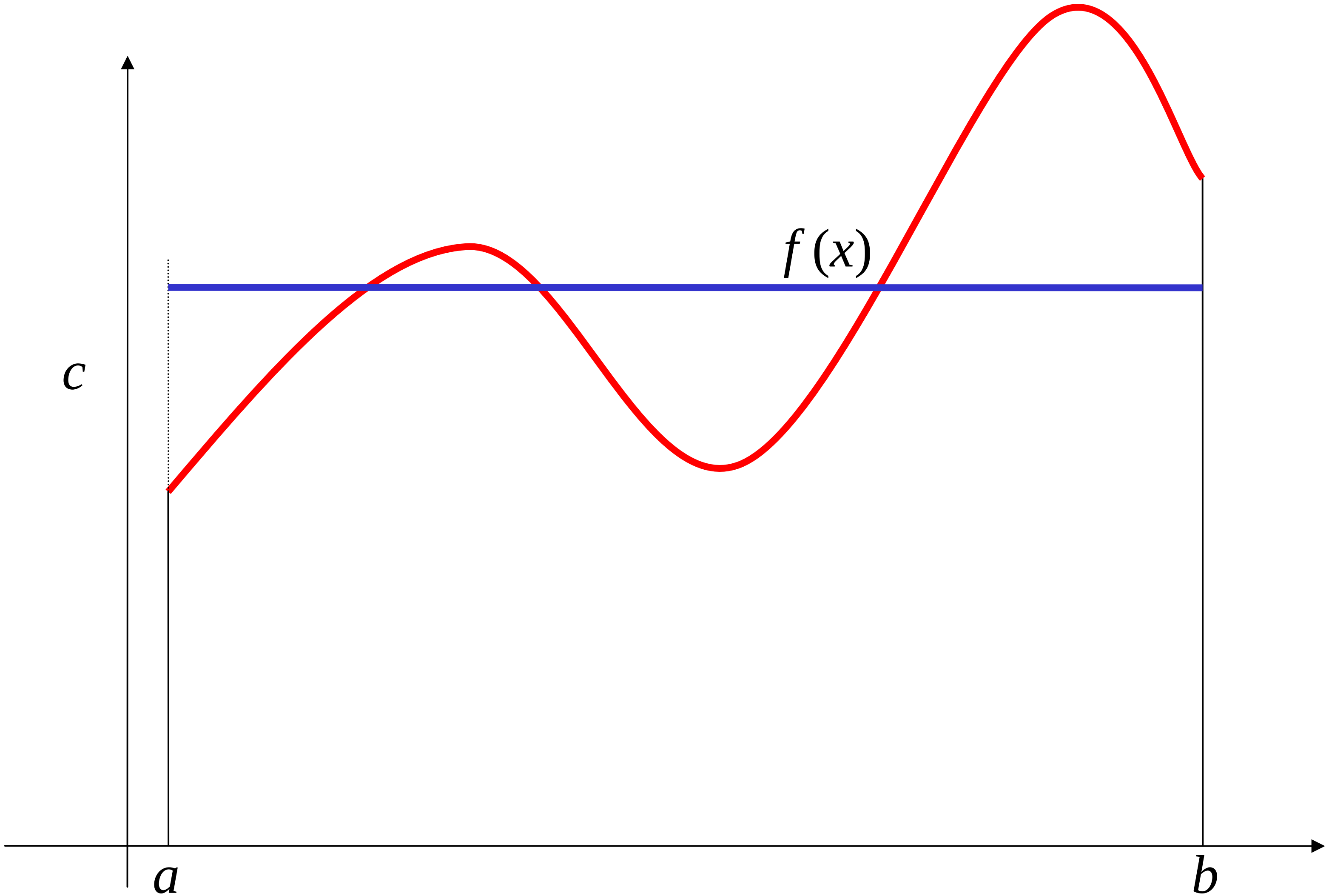
for some $c \in [a,b]$

Mean value theorem for definite integrals II

If $f(x)$ is continuous for $a \le x \le b$ then

$$\boxed{\int_a^b f(x)\,dx = f(c)(b-a)}$$

for some $c \in [a,b]$



$f(x)$
c
a
b