Functions of *n* variables

Let E_n be a Euclidian space and $f : E_n \to R$ a mapping.

We say that f is a function of *n* variables and write

$$y = f\left(x_1, x_2, \ldots, x_n\right)$$

If $X \in E_n$ and $X = \left[x_1, x_2, \ldots, x_n\right]$, we may also write

$$y = f\left(X\right)$$

The graph of a function of n variables

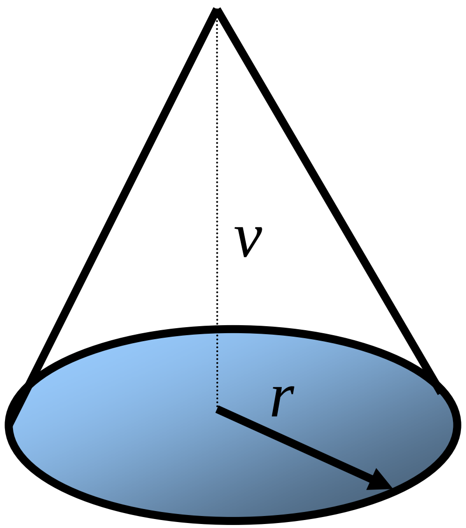
Let a function $y = f(x_1, x_2, \ldots, x_n)$ be given on a set $M \subseteq E_n$

We define the graph of $y = f(x_1, x_2, \ldots, x_n)$ as a set $G \subseteq E_{n+1}$

$$G = \{(x_1, x_2, \ldots, x_n, y) \mid (x_1, x_2, \ldots, x_n) \in M \wedge y = f(x_1, x_2, \ldots, x_n)\}$$

Note that the idea of a graph can only be utilized geometrically if $n = 1$ or $n = 2$. For higher dimensions it is of no practical significance

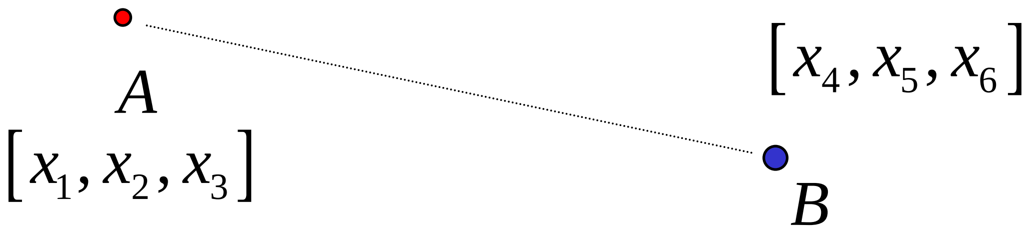
Examples



The volume of a cone

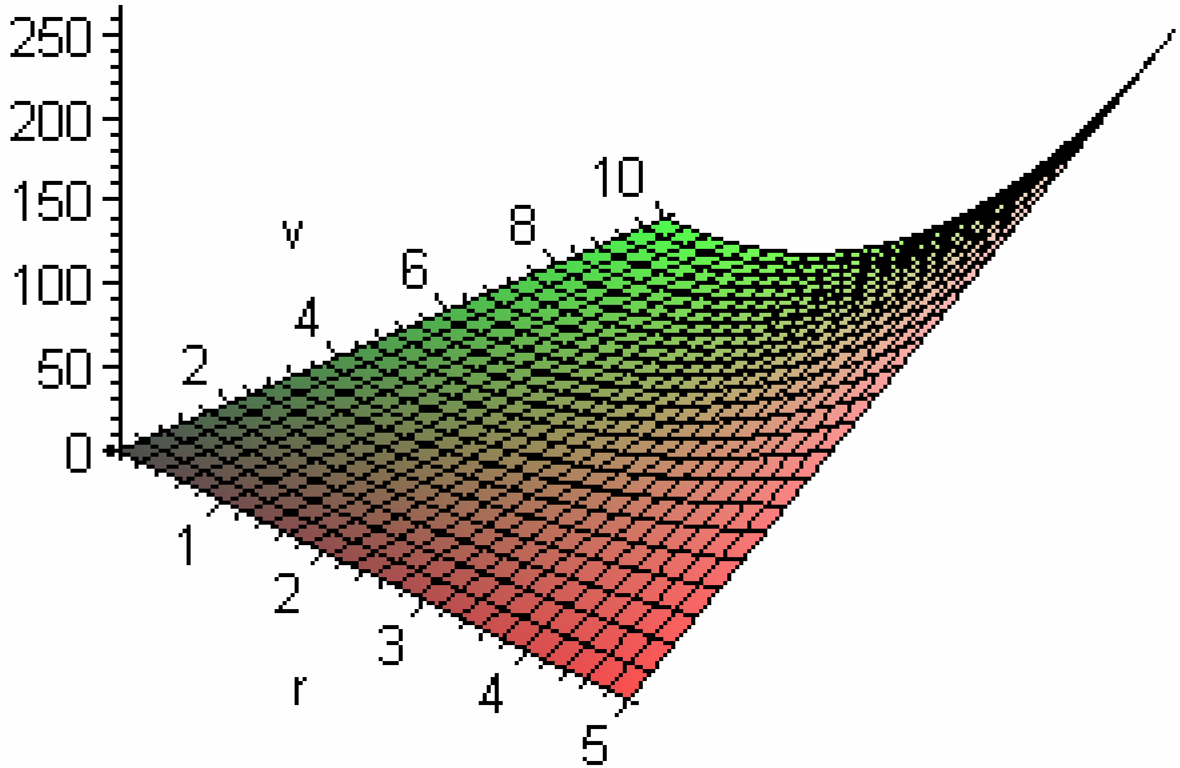
$$V(r,v)=\frac{\pi r^2 v}{3}$$

The distance of two points



$$\rho(A,B)=d(x_1,x_2,x_3,x_4,x_5,x_6)=\sqrt{\sum_{i=1}^{3}(x_{i+3}-x_i)^2}$$

$$V(r,v) = \frac{\pi r^2 v}{3}$$



There are properties of functions of *n* variables (or *n*-functions) that can be easily defined by analogy with functions of one variable:

- boundedness
- sum, difference, product, and quotient of two functions
- multiplying a function by a scalar
- domain of a function

Composite function

Let $h(u_1, u_2, \ldots, u_m)$ be a function of m variables and let for $X \in E_n$

$X = [x_1, x_2, \ldots, x_n]$ $g_1(X), g_2(X), \ldots, g_m(X)$ be m functions

of n variables. If $U = \left[g_1(X), g_2(X), \ldots, g_m(X) \right] \in D(h)$

for $X \in D(g_1) \cap D(g_1) \cap \ldots \cap D(g_m)$, we can assign to every X

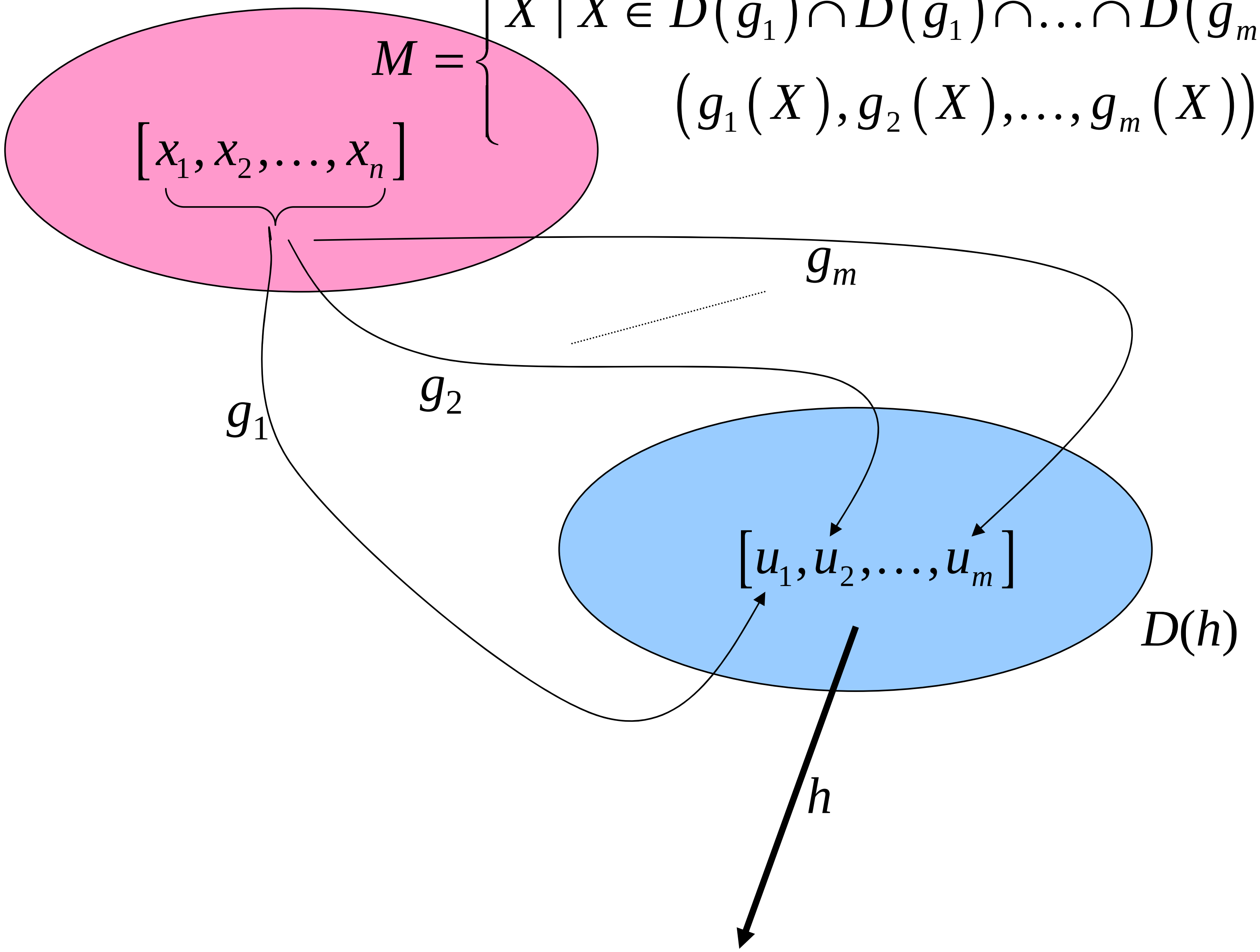
the value $y = h(U) = h\left(g_1(X), g_2(X), \ldots, g_m(X) \right)$ and define on

$$M = \left\{ \begin{array}{l} X \mid X \in D(g_1) \cap D(g_1) \cap \ldots \cap D(g_m) \wedge \\ \qquad\qquad \left(g_1(X), g_2(X), \ldots, g_m(X) \right) \in D(h) \end{array} \right\}$$

the function $f(X) = h\left(g_1(X), g_2(X), \ldots, g_m(X) \right)$ of n variables

composed of an outer component f and inner components $g_1, g_2, \ldots, g_n$

$$M = \left\{ X \mid X \in D(g_1) \cap D(g_1) \cap \ldots \cap D(g_m) \wedge \left(g_1(X), g_2(X), \ldots, g_m(X)\right) \in D(h) \right\}$$



$$f(X) = h\left(g_1(X), g_2(X), \ldots, g_m(X)\right)$$

Example

$$f(x,y,z)=\frac{xy+xz+yz}{xyz}$$

$$h(u_1,u_2)=\frac{u_1}{u_2}$$

$$g_1(x,y,z)=xy+xz+yz$$

$$g_2(x,y,z)=xyz$$

$$f(x,y,z)=h\left(g_1(x,y,z),g_2(x,y,z)\right)$$