<u>Local minimum</u>

Let $f\left(x_1, x_2, \mathbf{K}, x_n\right)$ be a function defined in an area containing

a point $A = \left[a_1, a_2, \mathbf{K}, a_n\right]$. If there is a neighbourhood $N\left(A, d\right)$

such that $f\left(X\right) \geq f\left(A\right)$ for all $X \in N\left(A, d\right)$ we say that

$f\left(x_1, x_2, \mathbf{K}, x_n\right)$ has a **local minimum** at A.

If $f\left(X\right) > f\left(A\right)$, then we say that the local minimum is **strict**

<u>Local maximum</u>

Let $f\left(x_1, x_2, \mathbf{K}, x_n\right)$ be a function defined in an area containing a point $A=\left[a_1, a_2, \mathbf{K}, a_n\right]$. If there is a neighbourhood $N\left(A, d\right)$ such that $f\left(X\right) \leq f\left(A\right)$ for all $X \in N\left(A, d\right)$ we say that $f\left(x_1, x_2, \mathbf{K}, x_n\right)$ has a **local maximum** at A.

If $f\left(X\right) < f\left(A\right)$, then we say that the local maximum is **strict**

What are the necesary conditions that a function

$f\left(x_1, x_2, \ldots, x_n\right)$ has to fulfil to have a local minimu or

maximum at a given point ?

Let $A=[a_1, a_2, \ldots, a_n]$ be an internal point of the domain of the function $f(x_1, x_2, \ldots, x_n)$ and let

$$\frac{\partial f(A)}{\partial x_i} \neq 0$$

for some *i*, $1 \le i \le n$. Then $f(x_1, x_2, \ldots, x_n)$ can have **neither** local **minimum nor** local **maximum** at *A*.

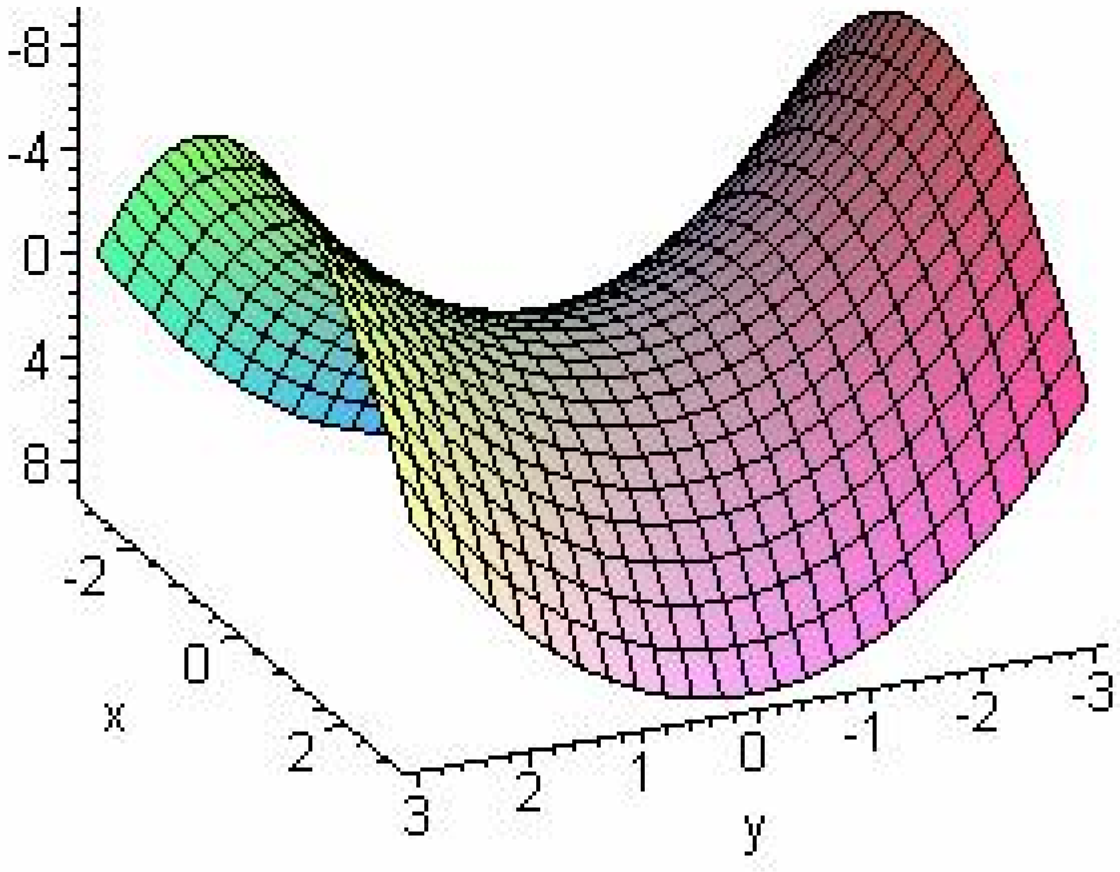
Note that, by the preceding theorem, a function can only have a local maxim or minimum at points where either all its partial derivatives vanish or those that do not vanish do not exist.

The condition

$$\frac{\partial f\left(a_1, a_2, \ldots, a_n\right)}{\partial x_1} = \frac{\partial f\left(a_1, a_2, \ldots, a_n\right)}{\partial x_2} = \cdots = \frac{\partial f\left(a_1, a_2, \ldots, a_n\right)}{\partial x_n} = 0$$

is by no means sufficient for a function to have a local maximum of minimum at $A = \left[a_1, a_2, \ldots, a_n\right]$ as the next example shows

Hyperbolic paraboloid $z = x^2 - y^2$ at $[x, y] = [1,1]$



What are the sufficient conditions that a function $f\left(x_1, x_2, \ldots, x_n\right)$ has to fulfil to have a local minimum or maximum at a given point ?

Two-function (maximum)

Let a function $f(x,y)$ have partial derivatives of second order continuous at $A=[a_1,a_2]$ and let $f_x'(A)=f_y'(A)=0$

Put $D(x,y)=\begin{vmatrix} f_{xx}''(x,y) & f_{xy}''(x,y) \\ f_{yx}''(x,y) & f_{yy}''(x,y) \end{vmatrix}$

If $D(a_1,a_2)>0$ and $f_{xx}''(a_1,a_2)<0$ then $f(x,y)$ has a strict local maximum at $A=[a_1,a_2]$

Two-function (minimum)

Let a function $f(x,y)$ have partial derivatives of second order continuous at $A=[a_1,a_2]$ and let $f_x^{'}(A)=f_y^{'}(A)=0$

Put $D(x,y)=\begin{vmatrix} f_{xx}^{"}(x,y) & f_{xy}^{"}(x,y) \\ f_{yx}^{"}(x,y) & f_{yy}^{"}(x,y) \end{vmatrix}$

If $D(a_1,a_2)>0$ and $f_{xx}^{"}(a_1,a_2)>0$ then $f(x,y)$ has a strict local minimum at $A=[a_1,a_2]$