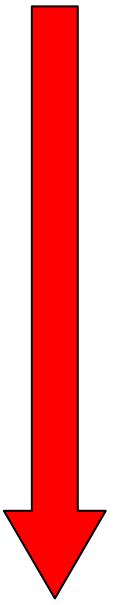## ODE with separable variables

$$y' = f(x)\,g(y)$$

$$\frac{dy}{dx} = f(x)\,g(y)$$

$$\frac{dy}{g(y)} = f(x)\,dx$$

$$\int \frac{dy}{g(y)} = \int f(x)\,dx + c$$

Examples

Find the general solution

$$xy' - y = y^2$$

$$\frac{dy}{dx}\frac{x}{} - y = y^2$$

$$\int \frac{dy}{y^2 + y} = \int \frac{dx}{x}$$

$$\int \left(\frac{1}{y} - \frac{1}{y+1} \right) dy = \int \frac{dx}{x}$$

$$\ln \left| \frac{y}{y+1} \right| = \ln|x| + c$$

$$\frac{y}{y+1} = cx$$

$$y = \frac{cx}{1 - cx}$$

Examples

Find the general solution

$$\tan x \, \sin^2 y \, dx + \cos^2 x \cot y \, dy = 0$$

$$\int \frac{\tan x}{\cos^2 x} dx = -\int \frac{\cot y}{\sin^2 y}$$

$$\frac{\tan^2 x}{2} = \frac{\cot^2 y}{2} + c$$

$$y = \text{arc} \cot \sqrt{\tan^2 x + c*}$$

Example

Find the particular solution to the following ODE passing through the point [0,1]

$$\left(1+e^x\right)yy'=e^x$$

$$1=y(0)=\sqrt{\ln(4c)}$$

$$\left(1+e^x\right)y\frac{dy}{dx}=e^x$$

$$c=\frac{e}{4}$$

$$\int y\,dy=\int\frac{e^x}{1+e^x}dx$$

$$y=\sqrt{\ln\left[e\left(\frac{1+e^x}{2}\right)^2\right]}$$

$$\frac{1}{2}y^2=\ln\left|1+e^x\right|+c$$

$$y=\sqrt{\ln\left(c\left(1+e^x\right)^2\right)}$$

Example

Find the particular solution to the following ODE passing through the point [3,2]

$$y' = -\frac{y}{x}$$

$$3 = \frac{c*}{2}$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$y = \frac{6}{x}$$

$$\ln|y| = -\ln|x| + c$$

$$y = \frac{c*}{x}$$