Mean value of a discrete random variable

Let a discrete random variable X assume values from a set M and have probability function $p(x)$. We define the **mean value** or **expectancy** $E(X)$ as follows:

$$\boxed{E(X) = \sum_{x \in M} x\, p(x)}$$

Note that if M is infinite, the mean value may not exist.

Mean value of a continuous random variable

Let a continuous random variable X assume values from a set M and have density function $p(x)$. We define the **mean value** or **expectancy** $E(X)$ as follows:

$$E(X) = \int_M x\, f(x)\, dx$$

provided that the integral over M exists.

Properties of expectancy

If the random variable X has an expectancy $E\,(X)$, then the random variable $Y = cX$ with c a real constant has the expectancy $E\,(Y) = c\,E\,(X)$.

If the random variables $X_1, X_2, \ldots, X_n$ have expectancies $E(X_1), E(X_2), \ldots, E(X_n)$, then the random variable $Y = X_1 + X_2 + \cdots + X_n$ has the expectancy

$$E(Y) = E(X_1) + E(X_2) + \cdots + E(X_n)$$

Variance of a random variable

For a random variable X (discrete or continuous) with expectancy $E(X)$ we can define its variance $D(X)$ as follows:

$$D(X) = E([X - E(X)]^2)$$

This formula has another equivalent form:

$$D(X) = E(X^2) - [E(X)]^2$$

Thus the variance of a random variable is the difference between the expectancy of the square of X and the square of the expectancy of X.

Properties of variance

If $Y = c\,X$ with c a real constant, then $D(Y) = c^2 D(X)$.

If $X_1, X_2, \ldots, X_n$ are independent random variables and $Y = X_1 + X_2 + \cdots + X_n$, then

$$D(Y) = D(X_1) + D(X_2) + \cdots + D(X_n)$$

Standard deviation

If a random variable X has a variance $D(X)$, then we define its standard deviation as $\sqrt{D(X)}$. It is sometimes denoted σ

$$\boxed{\sigma = \sqrt{E(X^2) - [E(X)]^2}}$$

Quantiles

Given a random variable X with probability distribution $F(x)$ and a number p, we define a p-quantile x_p by the following formulas

$$\lim_{x \to x_p -} F(x) \leq p$$

$$\lim_{x \to x_p +} F(x) \geq p$$

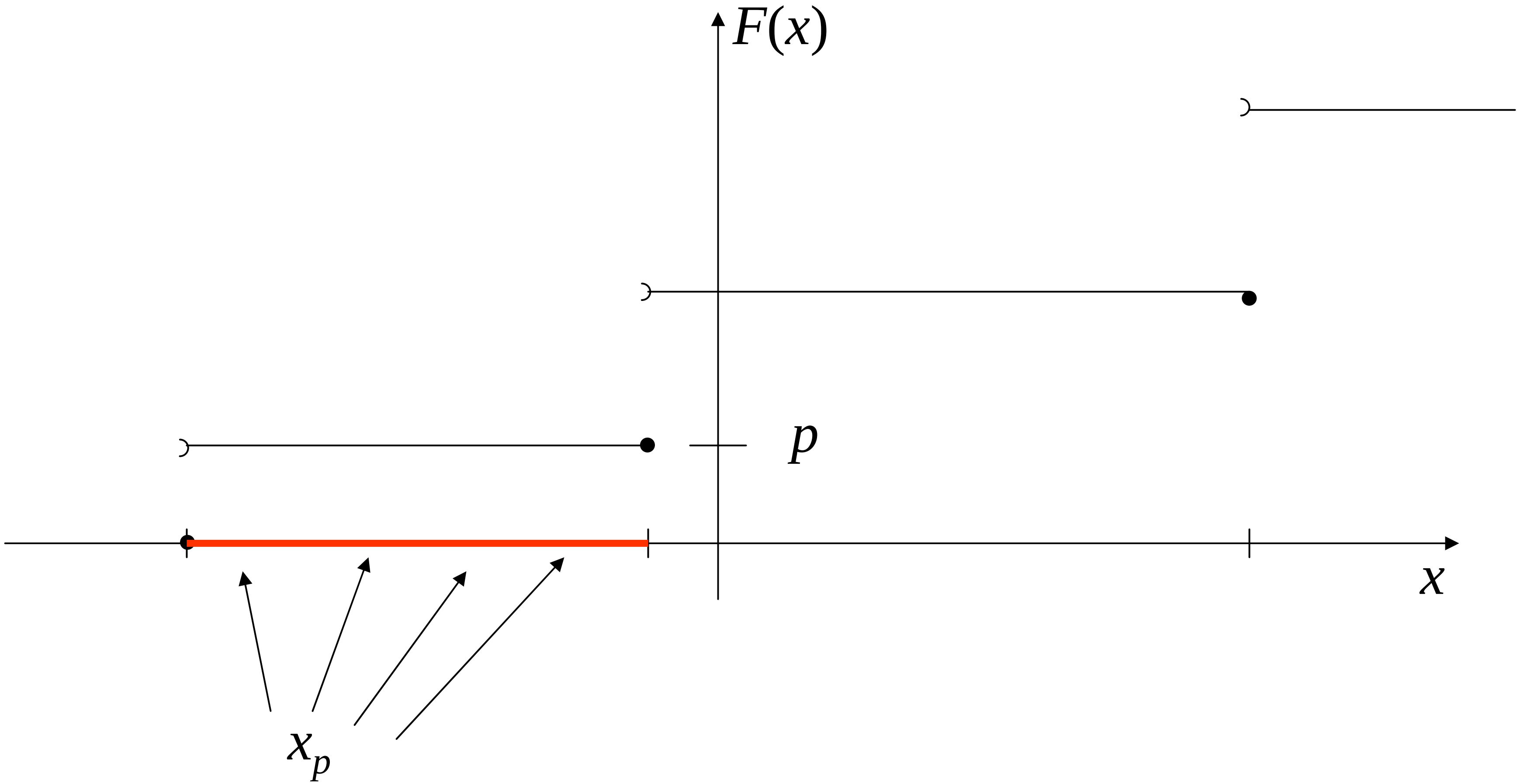
For a continuous increasing $F(x)$ a quantile is actually the inverse of $F(x)$

$$F(x_p) = p \qquad x_p = F^{-1}(p)$$

For a distribution $F(x)$ with jumps, we can determine quantiles as follows

$$\underset{x \to \alpha-}{F(x)} = p_1, \ \underset{x \to \alpha+}{F(x)} = p_2 \Rightarrow x_p = \alpha$$

for any $p_1 < p < p_2$



In intervals where the distribution is constant, the corresponding quantile is not determined uniquely

Special quantiles

$x_{0.01}$ - percentile: $x_{0.35}$ - 35-th percentile

$x_{0.25}$ - first quartile

$x_{0.50}$ - second quartile = median

$x_{0.75}$ - third quartile

Let, for a discrete random variable, the probability distribution be given by the probability function

x_i	x_1	x_2	x_3	$\cdots$	x_n
p_i	p_1	p_2	p_3	$\cdots$	p_n

If $\sum_{i=1}^{k} p_i \leq \alpha$ and $\sum_{i=1}^{k+1} p_i \geq \alpha$ we take

$$f_\alpha = x_k + \left(x_{k+1} - x_k\right) \frac{\alpha - \sum_{i=1}^{k} p_i}{p_{k+1}}$$

Example

Calculate the median of a discrete random variable whose probability function is given by the following table

x_i	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3
p_i	0.12	0.25	0.18	0.01	0.15	0.2	0.04	0.05

We have 0.12 + 0.25 = 0.37 and 0.12 + 0.25 + 0.18 = 0.55 so that

$$x_{0.5} = 1.5 + (1.8 - 1.5)\frac{0.5 - 0.37}{0.18} \approx 1.71667$$