PROBABILITY

Knowing everything about the circumstances of a future event, predict how likely that event is to happen?

STATISTICS

Studying the outcomes of several (many) accomplished events, find the maximum about the circumstances that govern the occurrence of such events

Poker is played with a standard 52-card deck in which all suits are of equal value, the cards ranking from the ace high, downward through king, queen, jack, and the numbered cards 10 to the deuce. The ace may also be considered low to form a straight (sequence) ace through five as well as high with king-queen-jack-10.

King of hearts

Ace of diamonds

Jack of spades

Queen of clubs

The traditional ranking of hands in a deal:

(1) *royal flush the ace-king-queen-jack-ten sequence of the same suit*

(2) *straight flush (five cards of the same suit in sequence)*

(3) *four of a kind, plus any fifth card*

(4) *full house (three of a kind and a pair)*

(5) *flush (five cards of the same suit)*

(6) *straight (five cards in sequence)*

(7) *three of a kind*

(8) *two pair*

(9) *one pair*

(10) *no pair, highest card determining the winner*

- There are 4 types of royal flush as there are four suits: hearts, diamonds, spades, and clubs.
- There are 2 598 960 different hands.
- The chances of receiving a royal flush hand when cards are dealt are 4 : 2 598 960.

So, knowing that we are playing poker, we can predict the likelihood or probability that a particular event, that is, for example, royal flush occurs.

In such a case, probabilistic methods are used in a straightforward manner so this is a typical problem of the probability theory.

Suppose we do not know which game we are playing and are trying to make a guess from the several hands (possibly a great deal of hands) that we receive.

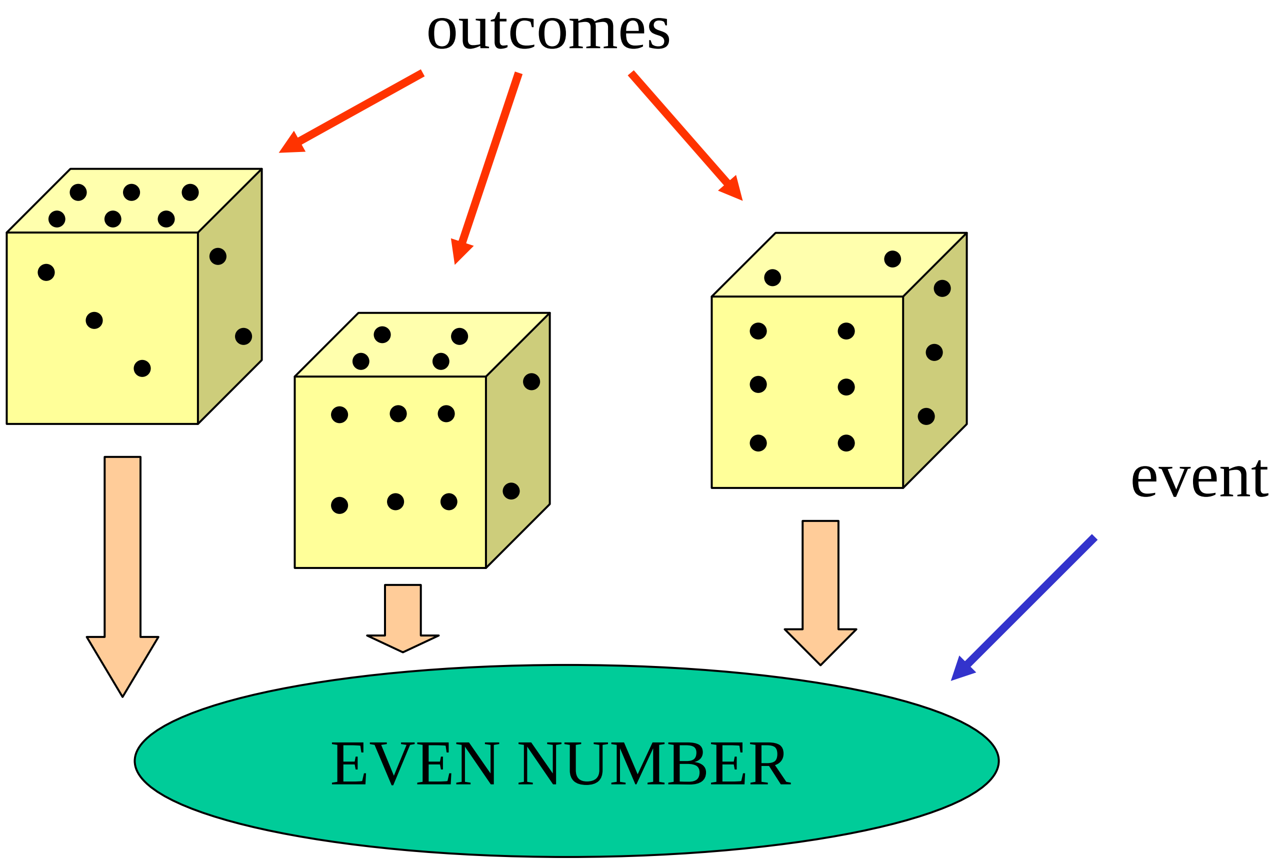
The hands may be thought of as samples of the population of playing cards. We analyse the samples and, using some tools of probability theory, draw a conclusion or make a statement about the nature of the game.

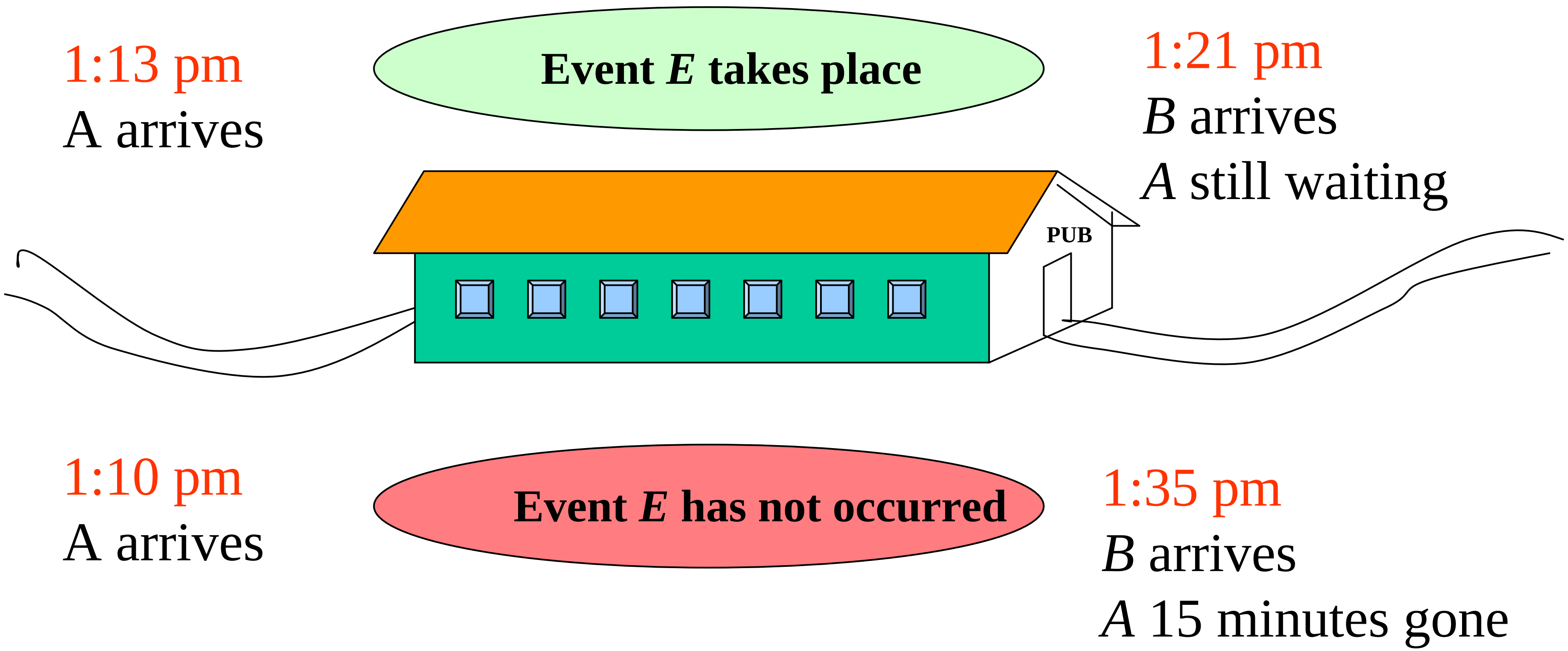
This final statement will, as a rule, involve probability, too, but the probability theory tools will be used in a more sophisticated way. This is typical of mathematical statistics.

EVENT

The fundamental notion of probability theory is an experiment that can be repeated, at least hypothetically, under essentially identical conditions and that may lead to different outcomes on different trials. The set of all possible outcomes of an experiment is called a **sample space**.

Sometimes we think of a set of outcomes as of an **event**. We say then that this event occurs whenever any of the set of outcomes occurs.


outcomes
event
EVEN NUMBER



Two persons *A* and *B* agree to meet in a place between 1 pm and 2 pm. However, they don't specify a more precise time. Each of them will wait for ten minutes and leave if the other doesn't turn up. By *E* we will denote the event that *A* and *B* really meet.

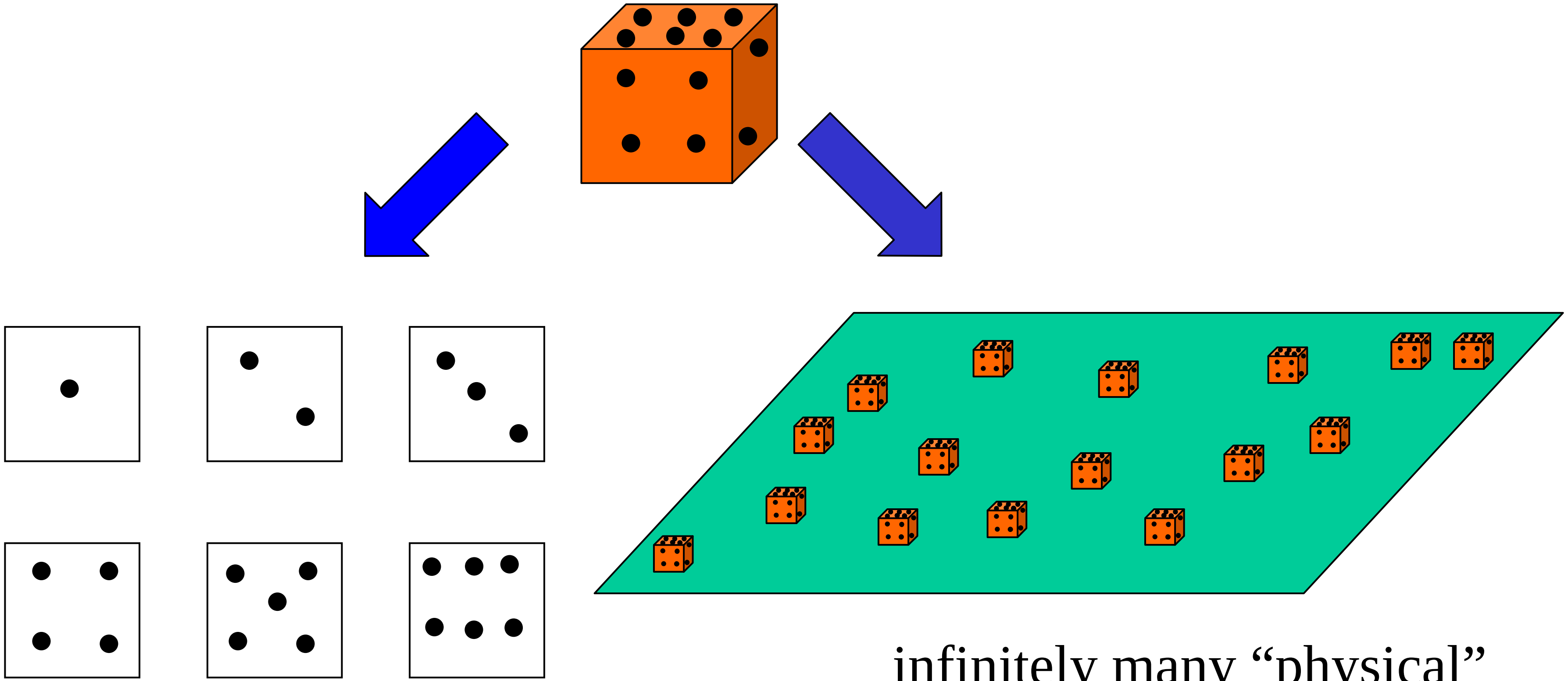
Sample space

$$\Omega = \{(t_A, t_B) \mid t_A, t_B \in [0,60]\}$$

Event

$$E = \left\{(t_A, t_B) \mid |t_A - t_B| \leq \frac{1}{6}\right\}$$

The way the outcomes of an experiment are conceived depends on particular circumstances. The “coarsest” distinction that still can describe all the subtleties of a particular problem is usually chosen.



infinitely many “physical” outcomes

6 “logical” outcomes

Mathematical model

An event E may be thought of as a subset of a sample space Ω

$$E \subseteq \Omega$$

With each event E is associated the complementary event $\overline{E}$ consisting of those experimental outcomes that do not belong to E.

$$\overline{E} = \Omega - E$$

For two events *A* and *B*, the ***intersection*** of *A* and *B* is the set of all experimental outcomes belonging to both *A* and *B* and is denoted

$$A \cap B$$

The ***union of events*** *A* and *B* is the set of all experimental outcomes belonging to *A* or *B* (or both) and is denoted .

$$A \cup B$$

The ***impossible event***, that is, the event containing no outcomes is denoted by **Ø** and the sample space Ω is sometimes called the ***sure event***.

Probability

- To an event E we assign a number called probability $P(E)$ as a measure of uncertainty or (certainty) of this event happening through one of the outcomes of an experiment.

- This probability may have different interpretations and several different methods of probability assigning may be devised .

- As a base we may take, for example, relative frequencies, for which simple games involving coins, cards, dice, and roulette wheels provide examples.

- Another interpretation of probability may be a personal measure of uncertainty.

Whichever method we choose, however, we should agree on some "natural" requirements and conventions that each definition of probability should meet.

AXIOMATIC DEFINITION

$$0 \le P(E) \le 1$$

$$P(\Omega) = 1$$

$$P(E_1 \cup E_2 \cup \ldots \cup E_n \cup \ldots) = P(E_1) + P(E_2) + \cdots + P(E_n) + \cdots$$

$$E_i \cap E_j = \varnothing \text{ if } i \neq j$$

Other important properties that any probability must have can be derived:

$$P(\overline{E}) = 1 - P(E)$$

$$P(\varnothing) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$