

```

> restart;
S := unapply(a[0] + a[1]*s + a[2]*s^2+a[3]*s^3,s);
dS := unapply(diff(S(s),s),s);
d2S := unapply(diff(dS(s),s),s);

```

$$S := s \rightarrow a_0 + a_1 s + a_2 s^2 + a_3 s^3$$

$$dS := s \rightarrow a_1 + 2 a_2 s + 3 a_3 s^2$$

$$d2S := s \rightarrow 2 a_2 + 6 a_3 s$$

Soustava (3.18):

```

> sol1:=solve({S(0)=y[i-1], S(h[i])=y[i], dS(0)=d[i-1],
dS(h[i])=d[i]},{a[0],a[1],a[2],a[3]});

```

$$\text{sol1} := \left[ \left[ \begin{array}{l} a_0 = y_{i-1}, a_1 = d_{i-1}, a_2 = -\frac{2 d_{i-1} h_i + 3 y_{i-1} - 3 y_i + h_i d_i}{h_i^2}, \\ a_3 = \frac{2 y_{i-1} + d_{i-1} h_i - 2 y_i + h_i d_i}{h_i^3} \end{array} \right] \right]$$

Soustava (3.27) podmínky:

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> sol2:=solve({S(0)=y[i-1], S(h[i])=y[i], d2S(0)=M[i-1],
d2S(h[i])=M[i]},{a[0],a[1],a[2],a[3]});

```

$$\text{sol2} := \left[ \left[ \begin{array}{l} a_0 = y_{i-1}, a_1 = -\frac{1}{6} \frac{6 y_{i-1} + 2 M_{i-1} h_i^2 + h_i^2 M_i - 6 y_i}{h_i}, a_2 = \frac{1}{2} M_{i-1}, a_3 = -\frac{1}{6} \frac{M_{i-1} - M_i}{h_i} \end{array} \right] \right]$$

Odvození vzorce (3.19):

```

> a[0]:=op(2,sol1[1][1]); a[1]:=op(2,sol1[1][2]);
a[2]:=op(2,sol1[1][3]); a[3]:=op(2,sol1[1][4]);

```

$$a_0 := y_{i-1}$$

$$a_1 := d_{i-1}$$

$$a_2 := -\frac{2 d_{i-1} h_i + 3 y_{i-1} - 3 y_i + h_i d_i}{h_i^2}$$

$$a_3 := \frac{2 y_{i-1} + d_{i-1} h_i - 2 y_i + h_i d_i}{h_i^3}$$

```
> a[2]:=collect(a[2],h[i]);
a[2]:=subs(-3*y[i-1]+3*y[i]=3*delta[i]*h[i],a[2]);
a[2]:=collect(a[2],h[i]);
```

$$a_2 := \frac{-2 d_{i-1} - d_i}{h_i} + \frac{-3 y_{i-1} + 3 y_i}{h_i^2}$$

$$a_2 := \frac{-2 d_{i-1} - d_i}{h_i} + \frac{3 \delta_i}{h_i}$$

$$a_2 := \frac{-2 d_{i-1} - d_i + 3 \delta_i}{h_i}$$

```
> a[3]:=collect(a[3],h[i]);
a[3]:=subs(2*y[i-1]-2*y[i]=-2*delta[i]*h[i],a[3]);
a[3]:=collect(a[3],h[i]);
```

$$a_3 := \frac{d_{i-1} + d_i}{h_i^2} + \frac{2 y_{i-1} - 2 y_i}{h_i^3}$$

$$a_3 := \frac{d_{i-1} + d_i}{h_i^2} - \frac{2 \delta_i}{h_i^2}$$

$$a_3 := \frac{d_{i-1} + d_i - 2 \delta_i}{h_i^2}$$

Vzorec (3.19)

```
> 'S[i]'=S(s);
```

$$S_i = y_{i-1} + d_{i-1} s + \frac{(-2 d_{i-1} - d_i + 3 \delta_i) s^2}{h_i} + \frac{(d_{i-1} + d_i - 2 \delta_i) s^3}{h_i^2}$$

Odvození vzorce (3.22)

```
> pom0 := collect(d2S(0),h[i]):
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pom1 := collect(d2S(h[i]),h[i]):
pom1 := subs(12*y[i-1]-12*y[i]=-12*delta[i]*h[i],pom1):
pom1 := collect(pom1,h[i]):
'diff(S[i],x$2)(x[i-1])'=pom0;
'diff(S[i],x$2)(x[i])'=pom1;
pom2:=subs(i=i+1,pom0):
'diff(S[i+1],x$2)(x[i])'=pom2;
pom3:=expand(1/2*h[i]*h[i+1]*simplify(pom1 =
pom2),delta[i],delta[i+1]):
pom3:=pom3 + 3*h[i+1]*delta[i]+2*h[i]*d[i]+h[i]*d[i+1]: pom3;

```

$$\left(\frac{\partial^2}{\partial x^2} S_i\right)(x_{i-1}) = \frac{-4 d_{i-1} - 2 d_i + 6 \delta_i}{h_i}$$

$$\left(\frac{\partial^2}{\partial x^2} S_i\right)(x_i) = \frac{2 d_{i-1} + 4 d_i - 6 \delta_i}{h_i}$$

$$\left(\frac{\partial^2}{\partial x^2} S_{i+1}\right)(x_i) = \frac{-4 d_i - 2 d_{i+1} + 6 \delta_{i+1}}{h_{i+1}}$$

$$h_{i+1} d_{i-1} + 2 h_{i+1} d_i + 2 h_i d_i + h_i d_{i+1} = 3 h_i \delta_{i+1} + 3 h_{i+1} \delta_i$$

Odvozeni vzorce (3.27):

```

> a[0]:=op(2,sol2[1][1]); a[1]:=op(2,sol2[1][2]);
a[2]:=op(2,sol2[1][3]); a[3]:=op(2,sol2[1][4]);

```

$$a_0 := y_{i-1}$$

$$a_1 := -\frac{1}{6} \frac{6 y_{i-1} + 2 M_{i-1} h_i^2 + h_i^2 M_i - 6 y_i}{h_i}$$

$$a_2 := \frac{1}{2} M_{i-1}$$

$$a_3 := -\frac{1}{6} \frac{M_{i-1} - M_i}{h_i}$$

```

> a[1]:=collect(a[1],h[i]);
a[1]:=subs((-y[i-1]+y[i])/h[i]=delta[i],a[1]);

```

$$a_1 := \left(-\frac{1}{3} M_{i-1} - \frac{1}{6} M_i\right) h_i + \frac{-y_{i-1} + y_i}{h_i}$$

$$a_1 := \left(-\frac{1}{3} M_{i-1} - \frac{1}{6} M_i\right) h_i + \delta_i$$

Vzorec (3.27):

```

> 'S[i]'=S(s);

```

$$S_i = y_{i-1} + \left( \left( -\frac{1}{3} M_{i-1} - \frac{1}{6} M_i \right) h_i + \delta_i \right) s + \frac{1}{2} M_{i-1} s^2 - \frac{1}{6} \frac{(M_{i-1} - M_i) s^3}{h_i}$$

Vzorec (3.29)

```
[ > R:=simplify(6*dS(h[i])-6*subs(i=i+1,dS(0)))=0: R := R -
  (6*delta[i]-6*delta[i+1]): R;
      M_{i-1} h_i + 2 h_i M_i + 2 h_{i+1} M_i + h_{i+1} M_{i+1} = -6 \delta_i + 6 \delta_{i+1}
[ >
```