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[> restart;
[> P:=(t,x0,y0,x1,y1)->y0*(t-x1)/(x0-x1)+y1*(t-x0)/(x1-x0);

$$P := (t, x0, y0, x1, y1) \rightarrow \frac{y0(t - x1)}{x0 - x1} + \frac{y1(t - x0)}{x1 - x0}$$

[> Rich:=P(0,h^2/4,F(h/2),h^2,F(h));

$$Rich := \frac{4}{3} F\left(\frac{h}{2}\right) - \frac{1}{3} F(h)$$

[> P:=(t,x,y)->y[1]*(t-x[2])/(x[1]-x[2])+y[2]*(t-x[1])/(x[2]-x[1]);

$$P := (t, x, y) \rightarrow \frac{y_1(t - x_2)}{x_1 - x_2} + \frac{y_2(t - x_1)}{x_2 - x_1}$$

[> Rich:=P(0,[h^2/4,h^2],[F(h/2),F(h)]);

$$Rich := \frac{4}{3} F\left(\frac{h}{2}\right) - \frac{1}{3} F(h)$$

[> F2=subs(D(F)(0)=0,`@@~(D,3)(F)(0)=0,taylor(Rich,h=0,5));

$$F2 = F(0) - \frac{1}{96} (D^{(4)})(F)(0) h^4 + O(h^5)$$

[> Rich2:=P(0,[h^4/16,h^4],[F2(h/2),F2(h)]);

$$Rich2 := \frac{16}{15} F2\left(\frac{h}{2}\right) - \frac{1}{15} F2(h)$$

[> F3=subs(D(F2)(0)=0,`@@~(D,2)(F2)(0)=0,`@@~(D,3)(F2)(0)=0,`@@~(D,5)(F2)(0)=0,taylor(Rich2,h=0,7));

$$F3 = F2(0) - \frac{1}{14400} (D^{(6)})(F2)(0) h^6 + O(h^7)$$

[> taylor((f(x+h)-f(x-h))/(2*h),h=0);

$$D(f)(x) + \frac{1}{6} (D^{(3)})(f)(x) h^2 + \frac{1}{120} (D^{(5)})(f)(x) h^4 + O(h^5)$$

[> taylor((f(x+h)-2*f(x)+f(x-h))/(h^2),h=0,7);

$$(D^{(2)})(f)(x) + \frac{1}{12} (D^{(4)})(f)(x) h^2 + \frac{1}{360} (D^{(6)})(f)(x) h^4 + O(h^5)$$


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